

ENLARGING FIGURES BY DILATIONS- Student Handout

To rotate or translate an object, you can actually pick the object up and move it to its new location after the transformation. To reflect the object, you can at least turn it over and put it in the place it will be after the reflection.

1. Will the *shape* of the object ever change if you translate, rotate, or reflect it? Explain.
2. Will the *size* of the object ever change if you translate, rotate, or reflect it? Explain.
3. Sometimes people call translations, rotations, and reflections *rigid motions*. Do you think this is an appropriate name? Why or why not?

The following exploration is to introduce you to a kind of transformation that is not a rigid motion.

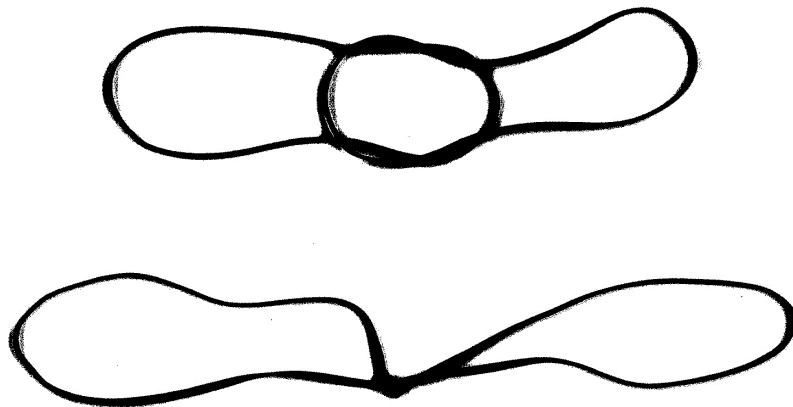
As you work on this exploration, think about these questions:

- Which features of a figure stay the same when it is enlarged?
- Which features change, and how?

Sarah had a stylized Bevo logo that she liked and wanted to put on the front of her ring binder. But it needed to be bigger, and she was having a hard time drawing a larger version. Her roommate Amy, who was student teaching in seventh grade math, said, "Oh, my students have just done a neat activity for enlarging a picture. Why not try it?" Here is the method Amy's students were using:

Instructions for using a "two-band stretcher":

- i. Make your "two-band stretcher" by taking two rubber bands of the same size (about 3"long) and linking them together, then pulling tight to make a "knot":



- ii. If you are right handed, tape the right-handed version of the picture to be copied to your desk and tape a blank paper on its right. (If you are left handed, use the left-handed version and tape the blank paper to its left.)
- iii. Use a finger of your non-writing hand to hold down one end of the stretcher on the point P. This point is called the *center of dilation*. (Amy's seventh graders call it the *anchor point*. Some people call it a *projection point*.)
- iv. Put a crayon or pencil (preferably not too sharp) in the other end of the stretcher. Stretch the stretcher with your crayon or pencil until the knot is on the outline you want to enlarge.
- v. Keeping the one end of the rubber band on point P, draw with your crayon or pencil so that the knot always stays on the outline to be enlarged. Keep your eye on the original picture, not on the one you are drawing. The resulting new picture is the *image* of the original picture under a *dilation*.

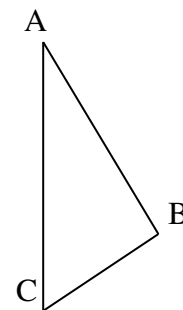
1. Use this method to enlarge the figures on Sheet 1 and Sheet 2. (Be sure to get the appropriate version of the sheet, depending on whether you are right- or left-handed.) *Note:* Don't be dismayed if your pictures are wiggly. You will develop a refinement of this procedure later.

2. Compare and contrast each original figure with its image under the dilation. (Ignore the imperfections; pretend you have drawn the images perfectly.) Be sure to consider all of the following:

- The lengths of the line segments.
- The areas of the figures.
- The angles.
- The general shape of the figure.

Be as detailed as possible. For example, don't just say that two lengths are different; tell as exactly as you can how they compare.

3. Copy triangle ABC onto a sheet of paper and choose your own center of dilation. Use that center of dilation and your stretcher to enlarge the triangle. Compare with other students' choice of center of dilation. Form a conjecture about what happens to the image when you change the center of dilation. Test out your conjecture (and if necessary, change it) using a center of dilation that is higher, one that is lower, one closer to the triangle, and one farther from the triangle.



4. Now you will figure out how to do the dilation without using the rubber bands. Draw another copy of triangle ABC on a new piece of paper. Pick a convenient center of dilation P (not too close, so the image doesn't overlap the original) *and look at the rubber band* as you make the image. Label your image as follows: Label as A' the point you draw when the knot is on A. Label B' and C' similarly. Thinking about the rubber band can help you answer the following questions:

- How do the lengths PA and PA' compare? How do you know without measuring?
- How do the lengths PB and PB' compare? How do you know without measuring?
- How do the lengths PC and PC' compare? How do you know without measuring?

Now you have the ideas you need to figure out how to use just a ruler and pencil to form a dilation image.

- Try it first with a new copy of triangle ABC. Pick your center of dilation P and use your ruler to find where points A', B', and C'. One you have these, you can draw in the triangle A'B'C'.
- Try it again with a new copy of triangle ABC, but this time pick the center of dilation P on the *right* side of the triangle if you are right-handed (or on the left side if you are left-handed.)
- Now use this method of finding an image under a dilation to make a neater enlargement of the Bevo logo than the one you made with the rubber band stretcher. Use Sheet 3, and see if you can pick your center of dilation so your image stays on the sheet.

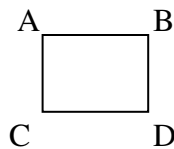
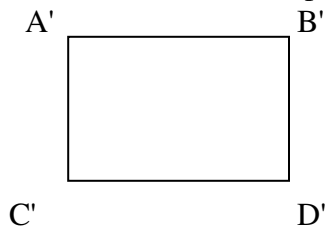
5. Now make a three-band stretcher by linking three identical rubber bands together. Use this stretcher to enlarge the drawings on Sheets 1 and 2, by keeping the knot closest to P on the original figure.

- How does the shape of the image compare to the shape of the original?
- How do the lengths of corresponding line segments in the original and the image compare?
- How do the areas of the image and original compare?

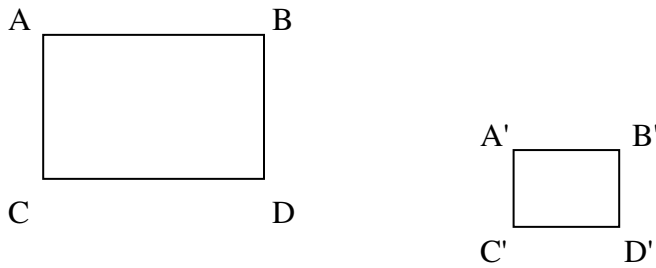
6. There are many possible dilations. Each dilation has an associated *center of dilation* and an associated *scale factor*. What do you think "scale factor" means? What is the scale factor for a dilation formed with a two-band stretcher? What is the scale factor for a dilation formed with a three-band stretcher?

Extensions:

1. Rectangle A'B'C'D' is the image of rectangle ABCD under a certain dilation. Find the center of dilation P. Explain how you found it and why your method works.



2. The *small* rectangle $A'B'C'D'$ is the *image of the large* rectangle $ABCD$ under a certain dilation. Find the center of dilation P . What can you say about the scale factor?



Note: A dilation like this, where the image is smaller than the original, is also called a *contraction*.

3. Triangle $A'B'C'$ has the same shape as triangle ABC , but can't possibly be the image of triangle ABC under any dilation. Explain why not. [Hint: Try to find a center of dilation.]



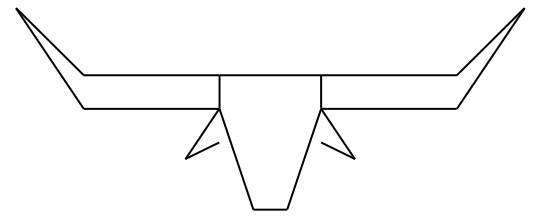
3. a. Find a rotation so that if you first transform triangle ABC by that rotation, then you can get triangle $A'B'C'$ from the transformed triangle by a dilation.

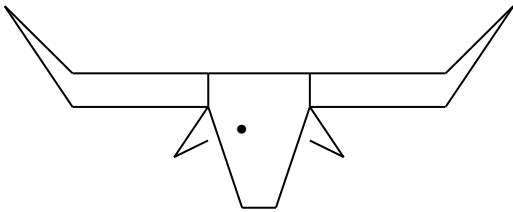


b. Find a reflection so that if you first transform triangle ABC by that reflection, then you can get triangle $A'B'C'$ from the transformed triangle by a dilation.



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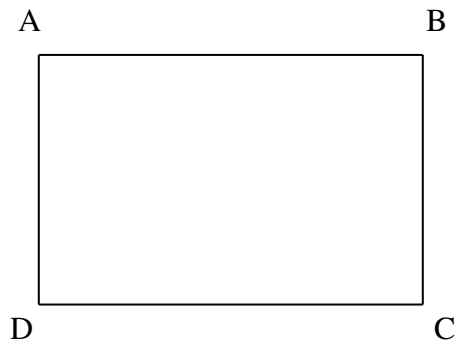




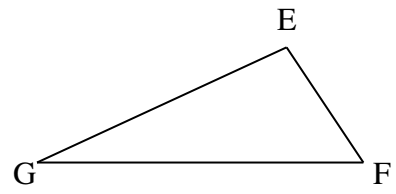
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SHEET 2 (Right-handed Version)

(Dilation Activity, M316L)

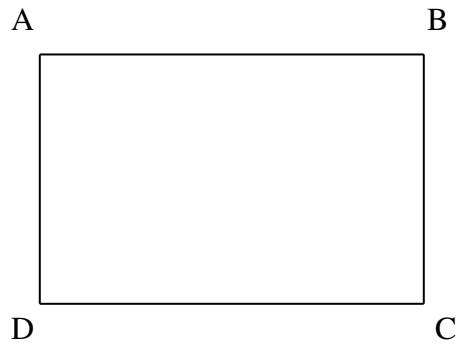


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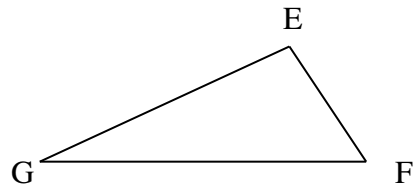


SHEET 2 (Left-handed Version)

(Dilation Activity, M316L)



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SHEET 3

(Dilation Activity, M316L)

