Fractal Exploration

M316L - Fall 2009

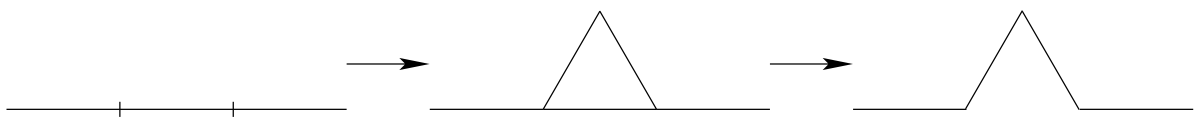
Part 1: Koch's Curve

Begin with a line segment, which we will say is 1 unit long:

Consider the following procedure:

1. Divide the segment into 3 pieces of equal length.
2. Using the middle piece as the base, construct an equilateral triangle pointing "upward".
3. Delete the base of the triangle obtained from step (2).

One application of this procedure is called an *iteration*. The procedure is illustrated below, and the resulting figure is on the right:



Note that there are now four congruent segments, whose lengths are each one third that of the original segment. For the next iteration, one would apply the procedure to each individual segment.

On the sheet provided, perform at least 4 iterations for yourself. Record data in the following table. [Hint: Use fractions.]

iteration	# of segments	segment length	total length
0	1	1	1
1	4	$\frac{1}{3}$	$\frac{4}{3}$

What are the patterns? Specifically, as we go from one iteration to the next. . .

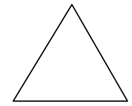
(a) How does the number of segments change?
(b) How does the length of the segments change?
(c) How does the total length change?

Finally, the total length grows with each new iteration.

(d) Can the total length become arbitrarily large? That is, with enough iterations, can it get larger than any number you can think of?

Part 2: Sierpinski's Triangle

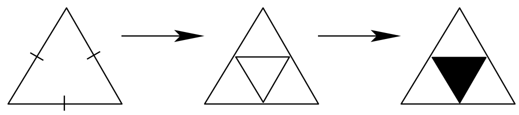
Begin with an equilateral triangle, whose sides we will say are each 1 unit long:



Consider the following procedure:

1. Find the midpoint of each side.
2. Connect all midpoints with line segments.
3. Delete the "middle" triangle.

The procedure is illustrated below, and the resulting figure is on the right:



Note that there are now three small triangles, whose side lengths are each half those of the sides of the original triangle, and whose area is one fourth that of the original triangle (a number which we denote by A). For the next iteration, one would apply the sequence to each individual small triangle.

Using the attached sheet, perform at least 4 iterations for yourself. Record data in the following table. In the column headings, "triangle" always means the smallest triangle at each step, and the "total area" is the area of what hasn't been deleted yet, written in terms of the original area A. [Hint: Again, use fractions.]

iteration	# of triangles	triangle side length	area of each triangle	total area
0	1	1	A	A
1	3	$\frac{1}{2}$	$\frac{1}{4}A$	$\frac{3}{4}A$

What are the patterns? Specifically, as we go from one iteration to the next...

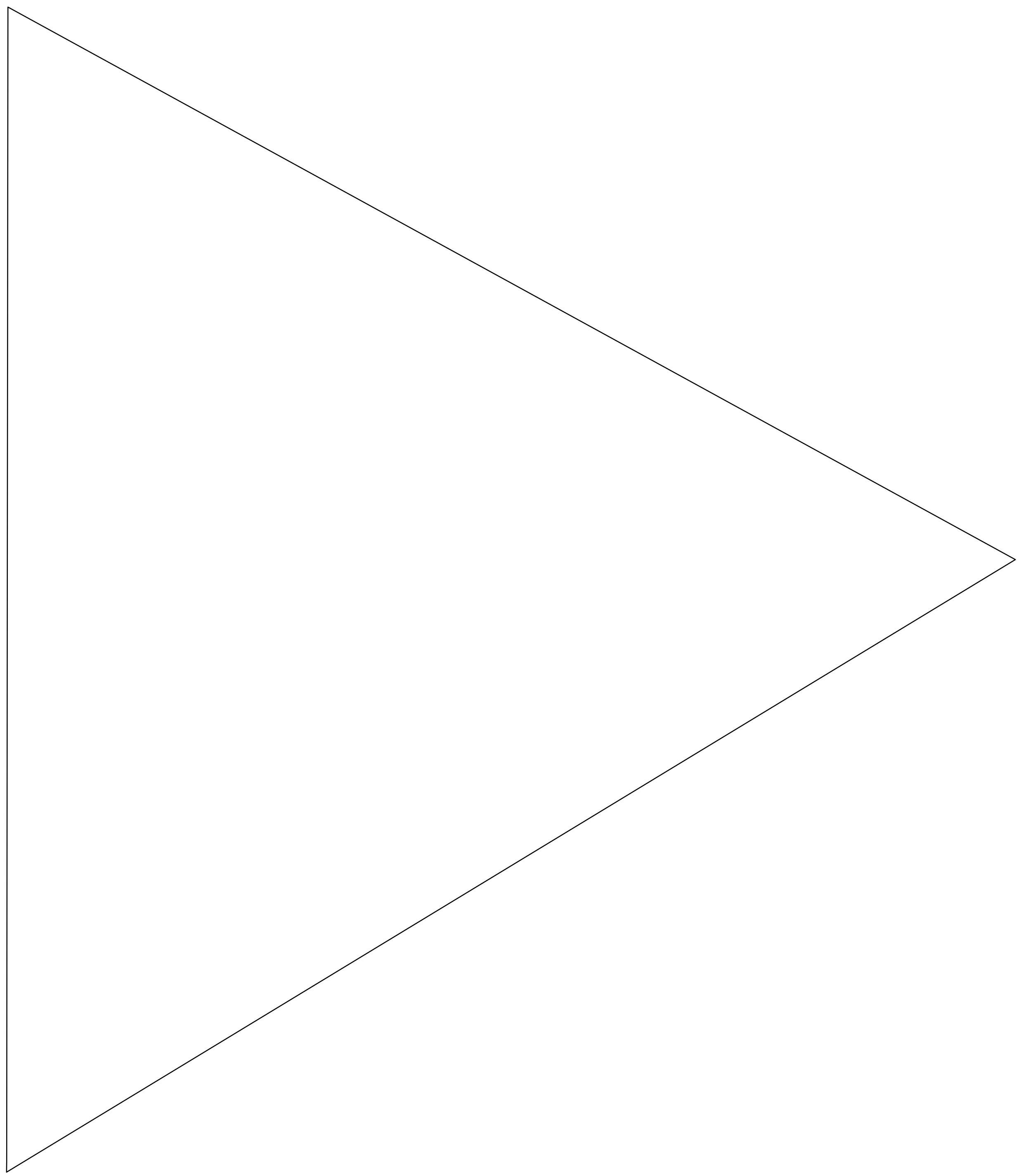
(a) How does the number of triangles change?
(b) How does the triangle side length change?
(c) How does the area of each triangle change?
(d) How does the total area change?

Finally, the total area shrinks with each new iteration.

(e) Can the total area get arbitrarily small? That is, with enough iterations, can it get smaller than any number you can think of?

Draw your Koch's curve on this page.

Draw your Sierpinski's Triangle on this page.



Part 3: Homework

Answer these questions on your own paper, unless otherwise indicated. This is due on Friday, October 30.

1. Be sure you complete Part 2 of the exploration (on Sierpinski's Triangle), but only turn in the picture.
2. (Koch's Snowflake) In the construction of Koch's curve, we could instead start with an equilateral triangle, and apply the same process to each side of the triangle. The result is a shape called *Koch's Snowflake*, which at each step is a polygon. Using the triangle on the back, perform at least 5 iterations, and record data in the following table. [Hint: Once again, use fractions.]

iteration	# of segments	segment length	perimeter
0	3	1	3
1	12	$\frac{1}{3}$	4

As the number of iterations increases, the total number of segments grows, the segment length shrinks, and the perimeter grows.

(a) How do the numbers in this table compare to the numbers in your table for Koch's Curve?

(b) After n iterations, the resulting figure is an N-gon. What is N? [This should be an expression involving n.]

(c) Can the perimeter get arbirarily large? That is, with enough iterations, can it get larger than any number you can think of?

(d) BONUS: The area of the polygon gets bigger as well. Can it get arbitrarily large? Why, or why not? [Hint: On what did you draw your snowflake?]

3. (Sierpinski vs. Pascal) You probably learned about Pascal's Triangle in M316K. Look at page 21 in your textbook to review it. Write down at least 20 rows of Pascal's triangle. [Suggestion: It may be easier to start from the corner of a blank sheet of paper.] Scratch out all of the even numbers. What do you see? How can you explain this?

Draw your Koch's Snowflake on this page.