M340L Second Midterm Exam, April 8, 2010

1. The matrices $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 12 \\ 1 & 0 & -2 & -1 & -3 \\ 0 & 2 & 6 & 5 & 37 \\ 2 & 1 & -1 & 2 & 23 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & -2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ are row-equivalent.

a) Find a basis for $Col(A)$. What is $dim(Col(A))$?

b) Find a basis for $Nul(A)$. What is $dim(Nul(A))$?

c) Find a basis for $Row(A)$. What is $dim(Row(A))$?

d) $M_{2,2}$ is the space of 2×2 matrices. Let V be the subspace of $M_{2,2}$ spanned by $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -2 \\ 6 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}, \begin{pmatrix} 12 & -3 \\ 37 & 23 \end{pmatrix} \right\}$. Find a basis for V.

2. On R^3, let $\mathcal{E}$ be the standard basis and let $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$.

Let $\mathbf{v} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$.

a) Compute the change-of-basis matrices $P_{\mathcal{E}\mathcal{B}}$ and $P_{\mathcal{B}\mathcal{E}}$

b) Compute $[\mathbf{v}]_{\mathcal{B}}$.

c) In P_2, let $\mathcal{C} = \{1+t+t^2, 2+3t+t^2, 1+t+2t^2\}$, and let $\mathbf{w} = 5-2t+3t^2$. Find $[\mathbf{w}]_{\mathcal{C}}$. (Justify your answer!)

3. Let $A = \begin{pmatrix} 6 & 5 \\ -5 & 0 \end{pmatrix}$ and let $B = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix}$.

a) Find the characteristic equation of A.

(b) Find the eigenvalues of A (you do not need to find the eigenvectors).

c) The eigenvalues of B are 1, 2 and 3. Find the corresponding eigenvectors. (Note: you may get some *simple* fractions in your calculations, but if you get any truly ugly denominators, you've made a mistake.)

4. a) Find a 2×2 matrix with eigenvalues 1 and 3, and with corresponding eigenvectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

(b) Is $\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$ diagonalizable? Why or why not?

5. True/false. Just mark each statement with a T (or TRUE) or an F (or FALSE). You do not need to justify your answers, and partial credit will not be given.

a) If a square matrix has determinant zero, then its null space is at least 1-dimensional.

b) The plane $x_1 + 2x_2 + 3x_3 = 6$ is a subspace of R^3.

c) If an $m \times n$ matrix has rank k, then its null space has dimension $m - k$.

d) If A is a 4×7 matrix, then the dimension of $Col(A)$ equals the dimension of $Row(A)$.

e) If $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are bases for a vector space V, then $P_{\mathcal{B}\mathcal{D}} = P_{\mathcal{C}\mathcal{D}} P_{\mathcal{B}\mathcal{C}}$.

f) The geometric multiplicity of an eigenvalue is at least as big as the algebraic multiplicity of that eigenvalue.

g) If the characteristic equation of a square matrix A is $(\lambda - 1)^3(\lambda + 2) = 0$, then $\lambda = 1$ is an eigenvalue with algebraic multiplicity 3.

h) If the characteristic equation of a real matrix A has complex roots, then there is no basis of R^n consisting of eigenvectors of A.

i) If $\mathcal{B}$ is a basis consisting of eigenvectors of A, then $[A]_{\mathcal{B}}$ is diagonal.

j) If A is a 5×5 matrix with eigenvalues 11, 25, 32 and 47, and if the geometric multiplicity of $\lambda = 11$ is 2, then A is diagonalizable.