Lecture 19 [highlights]

2 Nov 2010

Linear transform $T: V \to W$	$m \times n$ matrix A
$\mathrm{Ker}\, T \subset V$	$\mathrm{Nul}\, A \subset \mathbb{R}^n$
$\mathrm{Col}\, T \subset W$	$\mathrm{Ran}\, A \subset \mathbb{R}^m$
$\mathrm{Row}\, T \subset V$	———
$\dim \mathrm{Ker}\, T + \dim \mathrm{Col}\, T = \dim W$	$\dim \mathrm{Nul}\, A + \mathrm{rank}\, A = n$

Subspaces: Say H is a subset of V; H is a **subspace** of V if

- H contains the zero vector $\vec{0}$ of V
- H is closed under addition (if $\vec{h}_1$ is in H and $\vec{h}_2$ in H then $\vec{h}_1 + \vec{h}_2$ is in H)
- H is closed under scalar mult (if $\vec{h}$ in H and c any constant then $c\vec{h}$ is in H)

Cofactor:

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ -2 & 2 & 4 \end{vmatrix} = 0 \cdot \begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 3 \\ -2 & 2 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$= 0 - 1 \cdot (2+6) + 4 \cdot (1-6)$$

$$= 0 - 8 - 20 = \underline{\underline{-28}}$$

or:

$$\begin{matrix} 1 & 3 & 0 & 1 & 3 \\ 2 & 1 & 1 & 2 & 1 \\ -2 & 2 & 4 & -2 & 2 \end{matrix}$$

$$= (4 - 6 + 0) - (0 + 2 + 24)$$

$$= -2 - 26$$

$$= \underline{\underline{-28}}$$

(only for 3x3!)

$\dim \mathbb{P}_n = n+1$

$\dim \mathbb{R}^n = n$

An <u>invertible</u> $n \times n$ matrix has rank n

A <u>non-invertible</u> $n \times n$ matrix has rank $< n$

If A is an $m \times n$ matrix then $\dim \text{Row} A = \dim \text{Col} A = \text{rank} A$