Exercises marked with (⋆) will definitely not be graded.

**Exercise 1.** Guillemin/Pollack: Chapter 3, §3 (p. 116): 16 (to understand the hint you need to read pages 113-114), 17, 18, (19 (⋆), 20 (⋆)).

**Exercise 2.** Guillemin/Pollack: Chapter 3, §4 (p. 130): 2, 5, 10, 11.

**Exercise 3.** Describe (by formulas or by drawing a picture) a vector field on $S^2$ which has exactly three zeroes.

**Exercise 4.** Show that if $V$ is a complex vector space then the underlying real vector space $V_\mathbb{R}$ admits a canonical orientation.

(This is the beginning of a very nice story: if $M$ is a complex manifold then it also admits a canonical orientation, and this construction is automatically compatible with transverse intersections. In particular all intersection numbers turn out to be *nonnegative* for complex submanifolds intersecting transversally. However, the *self-intersection* number of a complex submanifold can be negative: this occurs when the transversal perturbations are not complex submanifolds anymore.)

**Exercise 5.** Show that the Lie bracket obeys Jacobi identity: for $X, Y, Z \in \mathfrak{X}(M)$,

$$[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0.$$  

**Exercise 6.** (⋆) Let $X, Y \in \mathfrak{X}(\mathbb{A}^n)$ compactly supported. Let $\phi_{t,X} : \mathbb{A}^n \to \mathbb{A}^n$ denote the flow of the vector field $X$ for time $t$. Show that

$$\lim_{t \to 0} \frac{1}{t^2} \left( \phi_{t,Y} \circ \phi_{t,X}(0) - \phi_{t,X} \circ \phi_{t,Y}(0) \right) = [X, Y](0).$$

(Hint: Taylor expand everything around 0.)