Nonabelian gauge theory

Now, replace U(1) by a general compact Lie group G:

\[ \mathcal{E} = \{ (P, V) \text{ G-bundle w/conn. on } X \} \quad G = \{ \text{equivalences } (P, V) \sim (P', V') \} \]

Fix invariant bilinear form (normalized to be 2 on the roots)

\[ S = \frac{1}{4g^2} \int_X \text{Tr}(F \wedge F) + \frac{\theta}{8\pi^2} \int_X \text{Tr}(F \wedge F) \]

Unlike the abelian theory, this is already interacting:

in triv. \( \nabla = d + A \), we have \( F = dA + [A, A] \)

so there are both cubic and quartic interactions.

Have to analyze it by RG methods like we did for the (abelian) scalar. \( \text{dim } A = 1 \).

\( \text{dim } F = 2. \)

Only to fix in \( \mathcal{G}/G \) with \( \text{dim } \leq 4 \), namely \( \text{Tr}(F \wedge F) \) and \( \text{Tr}(F \wedge F) \).

Now, a crucial difference this theory has \( \Lambda \frac{d \ln g}{d \Lambda} = - \frac{11N}{48\pi^2} g^3 + \ldots \)

(for \( G = SU(N) \))

\[ \text{(Nobel Prize 2004 Gross-Politzer-Wilczek)} \]

So, coupling gets weaker as \( \Lambda \) grows.

Perturbation theory gets more reliable. "asymptotic freedom".

Generally believed that in this case it's safe to take \( \Lambda_0 \to \infty \): the theory is "UV complete".

Conversely, coupling gets stronger at lower energies.

Let \( \Lambda_5 \) be the energy at which \( g = 1 \).

It's generally believed that the theory is actually \( \text{trivial at } E \ll \Lambda_5 \).

[Concrete consequence: e.g. \( \langle \text{Tr } F^2(x) \text{ Tr } F^2(0) \rangle \sim e^{-c \Lambda_5 x} \)]

Radically different from what we'd get in perturbation theory:

\[ D(x, 0) \sim \frac{1}{x^2} \]

No real proof even by physics standards. But, agrees w/ observation: we don't see SU(3) gauge fields around...
It is worth $1M from the Clay Foundation to rigorously (formulate and) prove this!

RG trajectories:

Can be fixed either by specifying $g$ at some fixed $E$, or by specifying the scale $\Lambda_s$. (The latter is usually preferred.)