Consider a gauge theory coupled to matter: \[ \mathcal{C} = \{(P, \nabla) : \text{principal } G-Lie \text{ w/con} \}
\]

\[ S = \frac{1}{4g^2} \int x F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} \int x |D\phi|^2 + \int x H(\phi) \]

where \( H \) is some \( G \)-invariant function.

Suppose moreover that \( H \) has minima other than the origin. For simplicity, say \( H \) has a single \( G \)-orbit worth of minima.

e.g. \( G = SU(2), \text{ } V = \text{fundamental, } H(\phi) = (||\phi||^4 - 2m^2 ||\phi||^2) \)

In this case, the perturbative physics at low energies will be best understood as an expansion around the orbit \( ||\phi|| = m \), not around \( \phi = 0 \).

In this pert. expansion, we can gauge-fix by taking \( P \) trivial and choosing say \( \phi = (1)_f \), free!. Then our theory is \( \approx \) the one with \( \mathcal{C} = \{ \text{conn. } A \text{ on } \text{triv. } P \}
\]

and no gauge group \( G \) anymore.

Write \( \phi_0 = m(1) \) and \( \phi = \phi_0 + \delta \phi \);

then \( |D\phi|^2 = |D(\phi_0 + \delta \phi)|^2 = |A\phi_0|^2 + (\text{terms involving } \delta \phi) \)

Let's see concretely what \( |A\phi_0|^2 \) looks like:

if \( A = A^1 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + A^2 \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + A^3 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ A^i \in \Omega^1(\mathcal{X}) \)

get \( |A\phi_0|^2 = m^2 (|A^1|^2 + |A^2|^2 + |A^3|^2) \)

i.e. all 3 components of \( A \) become massive. At energies \( E \ll gm \) they should be integrated out. \( \delta \phi \) is also massive.
If instead we take $V = \text{adjoint}$, then the picture is different: fix $\phi = (1 \quad 0 \quad 0 \quad 0 \quad 0)$, say, then we have a residual gauge symmetry left over:

$$G_{\text{eff}} = \text{Map} \{ X, \mathbb{Z}(\phi) \} \cong \text{Map} \{ X, U(1) \}$$

Have $\| [A, \phi_0] \|^2 = m^2 (\| A_2 \|^2 + \| A_3 \|^2)$, so only 2 of the 3 components of $A$ become massive. ("W bosons")

Set massive fields to 0 $\Rightarrow \quad S_{\text{eff}} = \frac{1}{4} \int F_A^2$

How reliable is this picture?

It would predict that the couplings run like:

$$g_m \begin{cases} \text{looks like } U(1) \text{ theory in this range} \\ \downarrow \text{ looks like } SU(2) \text{ theory in this range} \end{cases}$$

It will be most reliable when $m \gg \Lambda_s$, with $\Lambda_s$ the strong-coupling scale of the nonabelian theory. (In that case the effective coupling is weak everywhere along the flow.)

For $m \lessgtr \Lambda_s$, quantum corrections can change the picture even qualitatively.