Standard Model

Gauge theory with \( G = SU(3) \times SU(2) \times U(1) \)

Higgs field \( \phi \) charged under \( SU(2) \times U(1) \): if \( \phi \sim m(\phi) \) then
\[
Z_\phi = \left\{ (g, \left( e^{ix} \begin{pmatrix} 0 & \epsilon \nu \end{pmatrix} e^{-ix} \right) \} \subset SU(3) \times SU(2) \times U(1)
\]

Following the same procedure we followed in simpler examples here gives terms in \( S \) of the form (unprimed for \( SU(2) \), primed for \( U(1) \))
\[
\frac{m^2}{8} \left[ g^2 (2A^2 + \|A^2\|^2) + \|gA^3 - g'A^3\|^2 \right] + \frac{1}{2} \|F\|^2 + \frac{1}{2} \|F'\|^2
\]

Diagonalizing this then gives
\[
W^\pm = \frac{1}{\sqrt{2}} (A^+ \mp iA^-)
\]
\[
Z = \frac{1}{\sqrt{g^2 + g'^2}} (gA^3 - g'A')
\]

\[
m_W = \frac{1}{2} g m
\]
\[
m_Z = \frac{1}{2} \sqrt{g^2 + (g')^2} m
\]

\[
\sim 2 \cdot 10^{-33} \text{g}
\]

Measurements up to \( \sim \text{TeV} \) so far

\[
\begin{array}{cccccc}
\text{eV} & \text{KeV} & \text{MeV} & \text{GeV} & \text{TeV} \\
1 & 10^3 & 10^6 & 10^9 & 10^{12}
\end{array}
\]

Scales:
- electron mass \( 511 \text{ KeV} \)
- quark masses: \( u,d: \sim 1 \text{ MeV} \), \( s \sim 100 \text{ MeV} \), \( c \sim 1 \text{ GeV} \), \( b \sim 4 \text{ GeV} \), \( t \sim 170 \text{ GeV} \)
- weak scale: \( \sim 100 \text{ GeV} \) \( (W^\pm \sim 80 \text{ GeV} \), \( Z \sim 90 \text{ GeV} ) \)

At 100 GeV, \( \alpha_{SU(2)} \sim \frac{1}{30} \) \[
\alpha = \frac{g^2}{4\pi}
\]