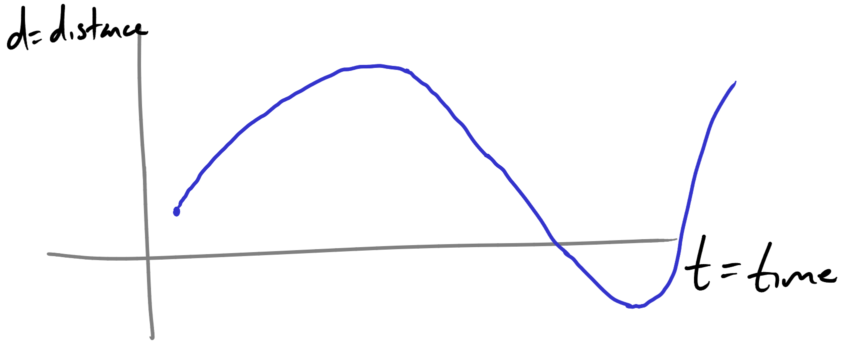
Lecture 4

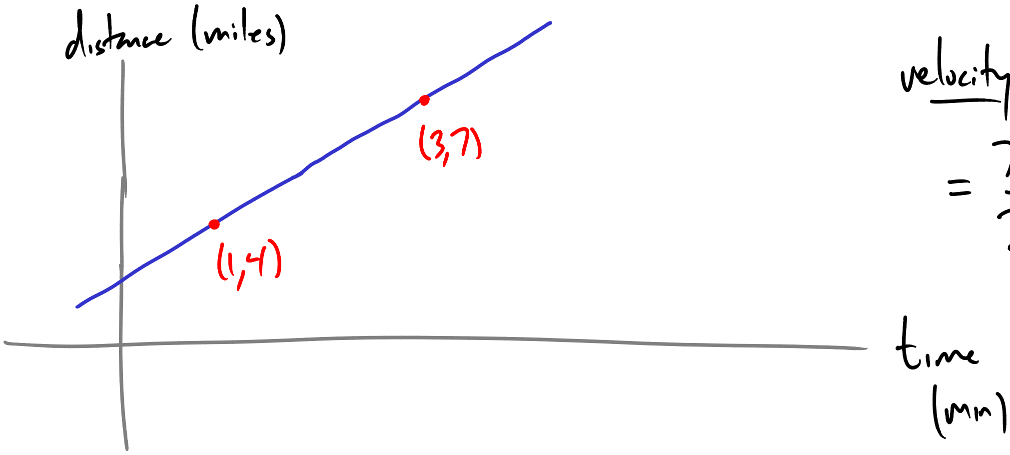
7 Sep 2018

My office hours: 2-3 M
4-5 W

Last time: inverse functions, logarithms

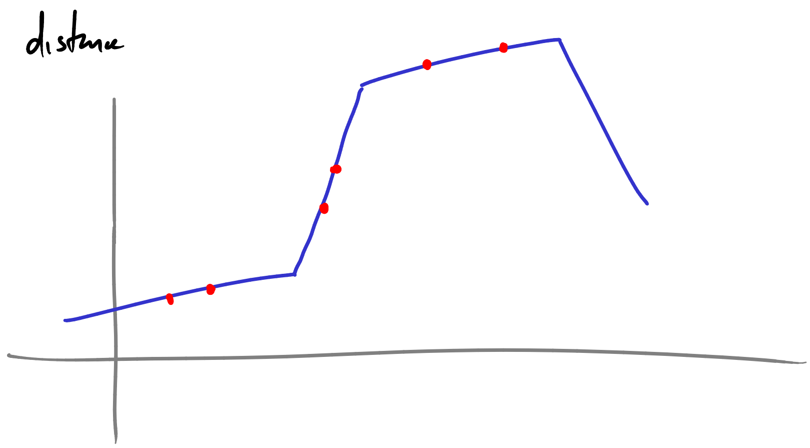


$f(t)$ = distance the object has traveled at time t

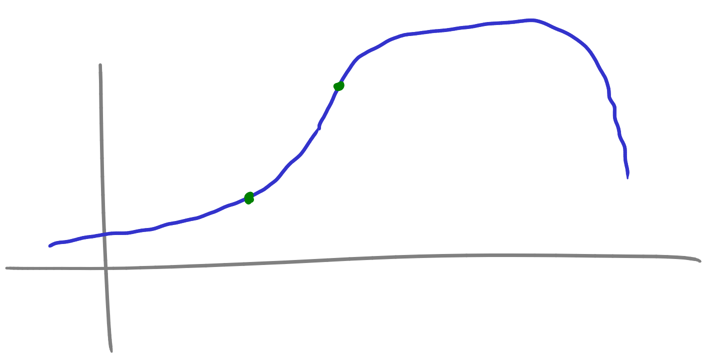


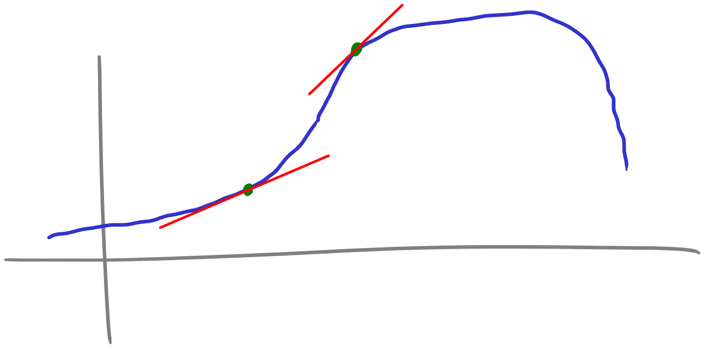
velocity is the slope of this line

$$= \frac{7-4 \text{ miles}}{3-1 \text{ min}} = \frac{3 \text{ mi}}{2 \text{ min}} = 1.5 \frac{\text{mi}}{\text{min}}$$

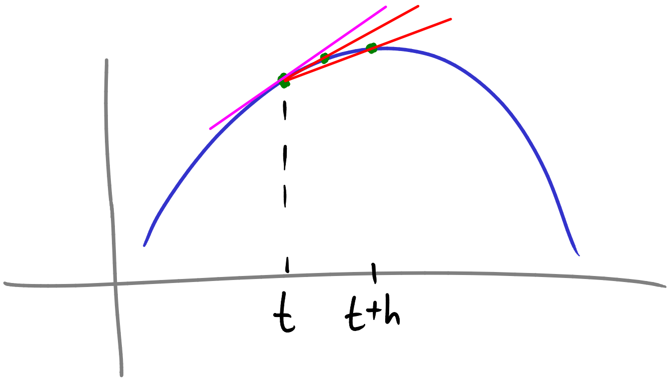


now have different speeds in the different parts, given by the slopes of the segments





Want to say velocity = slope of tangent line. How to make sense of that notion?



As we take h smaller and smaller the "secant lines" through

$(t, f(t))$ and $(t+h, f(t+h))$

are approaching a single fixed line — that's what we call the tangent line to the graph at $(t, f(t))$.

To formalize this, need the concept of limit.

Limits Suppose we have a function f, and as x gets close to a (but not equal to a) $f(x)$ gets close to L.

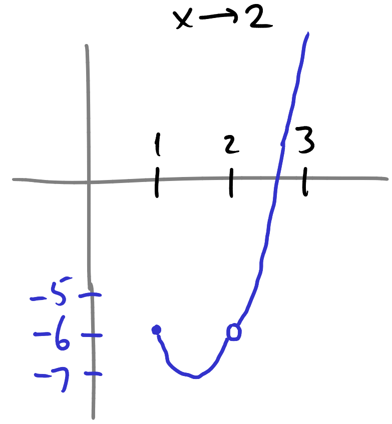
Then, we say $\lim_{x \to a} f(x) = L$.

Ex

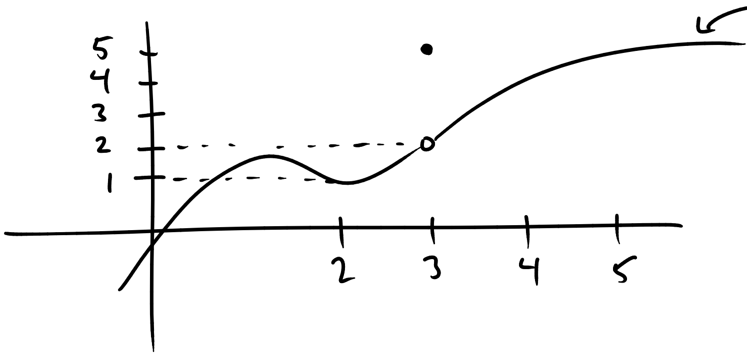
x	$f(x)$
1.0000	-6.000000
1.5000	-7.1250000
1.8000	-6.7680000
1.9000	-6.4410000
1.9900	-6.04940100
1.9990	-6.00499400
1.9999	-6.00049994
2.0001	-5.99949994
2.0010	-5.99499400
2.0100	-5.94939900
2.1000	-5.4390000
2.2000	-4.7520000
2.5000	-1.875000
3.0000	6.000000

2 is not in the domain of f

This table looks as if $\lim_{x \to 2} f(x) = -6$.



Ex



$$\lim_{x\to 3} f(x) = 2$$

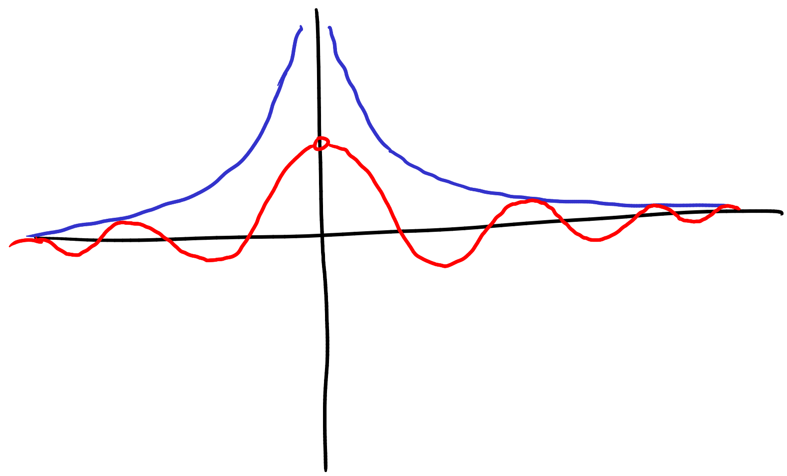
$$\lim_{x\to 2} f(x) = 1$$

Ex $f(x) = \dfrac{\sin x}{x}$

looks like $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = 1$

(we'll prove it later)

x	$f(x)$
-1.0000	0.8414710
-0.50000	0.9588511
-0.20000	0.9933467
-0.10000	0.9983342
-0.010000	0.9999833
-0.0010000	0.9999998
0.0010000	0.9999998
0.010000	0.9999833
0.10000	0.9983342
0.20000	0.9933467
0.50000	0.9588511
1.0000	0.8414710



Ex What is $\displaystyle\lim_{x\to 1} \frac{x-1}{x^3-1}$?

$\frac{1}{3}$, undef, 1

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x^2+x+1)}$$

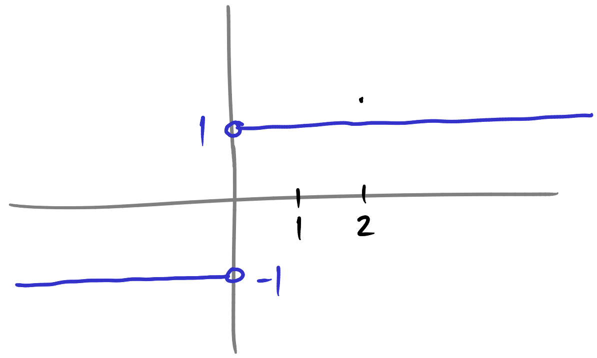
$$= \lim_{x\to 1} \frac{1}{x^2+x+1} = \frac{1}{1^2+1+1} = \frac{1}{3}$$

What is $\displaystyle\lim_{x\to 2} \frac{x-1}{x^3-1}$?

$$= \frac{2-1}{2^3-1} = \frac{1}{7}$$

Sometimes $\lim\limits_{x \to a} f(x)$ doesn't exist.

e.g. $f(x) = \frac{|x|}{x}$



$\lim\limits_{x \to 1} f(x) = 1$

but $\lim\limits_{x \to 0} f(x)$ does not exist.

One-sided limits: Suppose that as x gets close to a from the negative side (the left) $f(x)$ gets close to L. Then we say $\lim\limits_{x \to a^-} f(x) = L$.

$$\text{e.g. } \lim_{x \to 0^-} \frac{|x|}{x} = -1$$

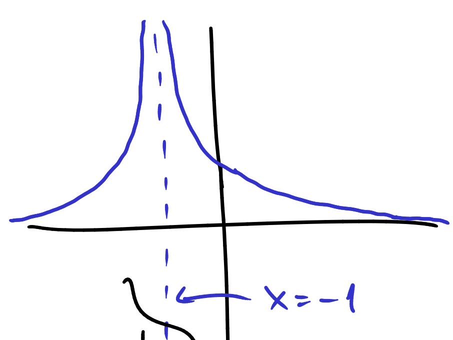
Similarly $\lim\limits_{x \to 0^+} \frac{|x|}{x} = +1$. $\quad \lim\limits_{x \to 2^-} \frac{|x|}{x} = 1$,

Limits of $\pm\infty$

Suppose as $x \to a$, $f(x)$ grows without bound in +ve direction.
Then we say $\lim\limits_{x \to a} f(x) = +\infty$.

Similarly if as $x \to a$, $f(x)$ grows w/o bound in −ve direction.
Then we say $\lim\limits_{x \to a} f(x) = -\infty$.

Ex $\lim\limits_{x \to -1} \frac{1}{(x+1)^2} = +\infty$



$\left[\text{if } x \text{ very close to } -1, \quad \frac{1}{(x+1)^2} = \frac{1}{(\text{very small})^2} = \text{very big} \right]$