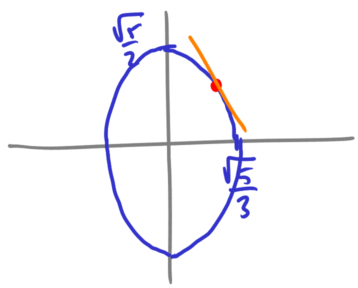
Lecture 14 | 3 Oct 2018

Midterm 1: grades should be available now
graded exams available by Friday

average: 95% whoa

Last time: implicit differentiation

Ex If $3x^2+2y^2=5$ then what are $y'(x)$, $y''(x)$ at $(x,y)=(1,1)$?



$$\frac{d}{dx}(3x^2+2y^2)=\frac{d}{dx}(5)=0$$

$$6x+4yy'=0 \quad (*) \qquad y'=\frac{dy}{dx}$$

$$y'=-\frac{6x}{4y}=-\frac{3x}{2y}$$

so at $(1,1)$
$$y'=-\frac{3}{2}$$

For y'': take $\frac{d}{dx}$ of $(*)$

$$\frac{d}{dx}(6x+4yy')=0$$

$$6+4y'y'+4yy''=0 \qquad \text{plug in: } x=1 \;\; y=1 \;\; y'=-\tfrac{3}{2}$$

$$6+4\left(-\tfrac{3}{2}\right)^2+4\cdot1\cdot y''=0$$

$$6+9+4y''=0$$

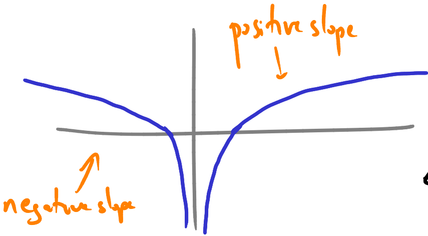
$$4y''=-15$$

$$y''=-\frac{15}{4}$$

Last time we used this method to find derivative of logarithm: $\frac{d}{dx}\ln x=\frac{1}{x}$ $(x>0)$

Q: What is $\frac{d}{dx}\ln|x|$?

domain: $(-\infty,0)\cup(0,\infty)$



← even function

$$\ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$$

So $\frac{d}{dx}\ln|x| = \begin{cases} \frac{d}{dx}\ln x = \frac{1}{x} & x>0 \\ \frac{d}{dx}\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x} & x<0 \end{cases}$

i.e. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

Can also use implicit diff. to get derivatives of <u>other inverse functions</u> eg inverse trig:

$\frac{d}{dx}\sin^{-1}x = ?$

Say $y = \sin^{-1}x$ we want $\frac{dy}{dx}$

$\sin y = x$ $\quad\quad \sin(\sin^{-1}x) = x$

$\frac{d}{dx}(\sin y) = \frac{d}{dx}(x)$

$(\cos y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

Since $y = \sin^{-1}x$ this means $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\cos(\sin^{-1}x)}$ — but that's a very ugly formula — want to <u>simplify</u>.

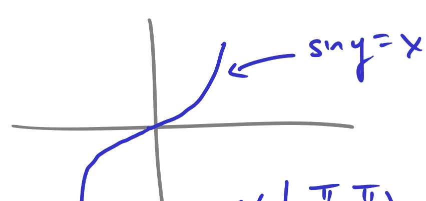
We know $\sin y = x$, want to express $\cos y$ in terms of x.

<u>Method 1</u>: know $\sin^2 y + \cos^2 y = 1$

$x^2 + \cos^2 y = 1$

$\cos^2 y = 1 - x^2$

So, $\cos y = \sqrt{1-x^2}$



$y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\rightarrow \cos y > 0$

So, $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\cos y} = \underline{\underline{\frac{1}{\sqrt{1-x^2}}}}$

Similarly

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

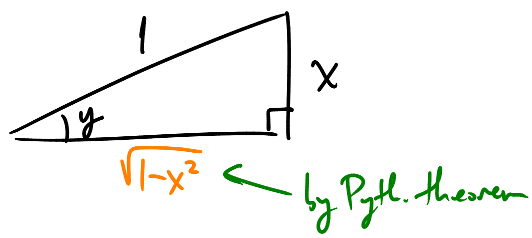
$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

<u>Method 2</u>: if we know $\sin y = x$ and want to express $\cos y$ in terms of x

draw a triangle:

$$x = \sin y = \frac{\text{opp}}{\text{hyp}}$$



$$\cos y = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

Similarly $\tan y = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{1-x^2}}$ etc.

<u>Ex</u> $y = x^x$ $\qquad \frac{dy}{dx} = ?$

<u>Not</u> power rule

Use $x = e^{\ln x}$

then $x^x = \left(e^{\ln x}\right)^x$

$$= e^{x \ln x}$$

So
$$\frac{d}{dx}\left(x^x\right) = \frac{d}{dx}\left(e^{x\ln x}\right)$$
$$= e^{x\ln x}\frac{d}{dx}(x\ln x)$$
$$= e^{x\ln x}\left(\ln x + \frac{x}{x}\right)$$

$$= e^{x\ln x}(\ln x + 1)$$

$$= \underline{\underline{x^x(\ln x + 1)}}$$