Housekeeping: Guest lectures next MW (Maria Guidelli)
No office hrs next M
Extra office hr next F (11-12 in add’l to usual 10-11)

Ex 2 April 6

Last time: iterated integrals
\[ \int_c^d \left[ \int_a^b f(x,y) \, dx \right] \, dy \]

= double integrals over rectangles

How about more complicated domains than rectangles?

Ex Evaluate \( \iint_D (x+2y) \, dA \) where \( D \) is the domain lying between the curves \( y=2x^2 \) and \( y=1+x^2 \).

It’s given by an iterated integral

\[ \int_{-1}^1 \left[ \int_{2x^2}^{1+x^2} (x+2y) \, dy \right] \, dx \]

\[ = \int_{-1}^1 \left[ xy + y^2 \right]_{y=2x^2}^{y=1+x^2} \, dx \]

\[ = \int_{-1}^1 \left[ x(1+x^2) + (1+x^2)^2 - (x(2x^2) + (2x^2)^2) \right] \, dx \]
We use this basic method whenever we have a domain of the form

\[ D = \{ (x,y) : a \leq x \leq b, \quad g_1(x) \leq y \leq g_2(x) \} \]

\[
\iint_D f(x,y) \, dA = \int_a^b \left[ \int_{g_1(x)}^{g_2(x)} f(x,y) \, dy \right] \, dx
\]

If \( D = \{ (x,y) : a \leq y \leq b, \quad h_1(y) \leq x \leq h_2(y) \} \)

\[
\iint_D f(x,y) \, dA = \int_a^b \left[ \int_{h_1(y)}^{h_2(y)} f(x,y) \, dx \right] \, dy
\]

\[ \text{Ex} \]

\[
\iint_D xy^2 \, dA
\]

\[ = \int_{-1}^{1} \left[ \int_0^\sqrt{1-y^2} xy^2 \, dx \right] \, dy
\]

\[ = \int_{-1}^{1} \left[ \frac{y^2 x^2}{2} \right]_{x=0}^{x=\sqrt{1-y^2}} \, dy
\]

\[ = \int_{-1}^{1} \left[ \frac{1}{2} y^2 (1-y^2) - 0 \right] \, dy \quad = \quad \cdots = \frac{2}{15}
\]

\( D \) is the domain enclosed by

- the line \( x=0 \)
- the circle \( x^2 + y^2 = 1 \)
- with \( x > 0 \)
\[ \int_0^1 \left( \int_x^1 \sin(y^2) \, dy \right) \, dx \]

This looks hard (can't do the inside \( \int \) over \( y \)).

Interpret it as a double \( \int \):

\[ \iint_D \sin(y^2) \, dA \]

where \( D \) is this triangle.

Do it by integrating over \( x \) first:

\[ \int_0^1 \left( \int_0^y \sin(y^2) \, dx \right) \, dy \]

\[ = \int_0^1 \left[ x \sin(y^2) \right]_{x=0}^{x=y} \, dy \]

\[ = \int_0^1 y \sin(y^2) \, dy \]

\[ = -\frac{1}{2} \cos(y^2) \bigg|_{y=1}^{1=1} \]

\[ = -\frac{1}{2} \cos(1) \bigg|_{y=0}^{y=0} \]

\[ = \frac{1}{2} (1 - \cos(1)) \]