 Lecture 2

Administration:

HW 01 due today, 3am
HW 02 due next Tue 3am

Learning Modules due every MW, Sat night (midnight)

We'll drop lowest 6 QR problems

My office Mon, 2-3 pm
W afternoons

Tom Cabello M 5-6 pm

Q 1) is \( \int (1+\sin x)^3 \, dx = \frac{1}{4} (1+\sin x)^4 + C \)? No

2) is \( \int x^7 \, dx = \frac{1}{8} x^8 + C \)? Yes

\( \int x^{-\frac{1}{3}} \, dx = \frac{2}{3} x^{\frac{2}{3}} + C \)? Yes

Some useful antiderivatives — see table on p. 403 of text

\[
\begin{align*}
\int e^x \, dx &= e^x + C \\
\int \sec^2 x \, dx &= \tan x + C \\
\int \sec x \tan x \, dx &= \sec x + C \\
\int \csc x \cot x \, dx &= -\csc x + C \\
\int \csc^2 x \, dx &= -\cot x + C \\
\int \frac{1}{\sqrt{1-x^2}} \, dx &= \arcsin x + C \\
\int \frac{1}{1+x^2} \, dx &= \arctan x + C
\end{align*}
\]
1) What is \( \int u^{3/2} \, du \)? The most general antiderivative is:

\[
\frac{3}{5} u^{5/3} + C
\]

2) Find a function \( F(u) \) with \( \frac{dF}{du} = u^{3/2} \) and \( F(1) = 1 \).

\[
\begin{align*}
\frac{dF}{du} &= u^{3/2} \\
F(u) &= \frac{3}{5} u^{5/3} + C \\
\text{and } F(1) &= 1, \text{ so} \\
\frac{3}{5} + C &= 1, \text{ so} \\
C &= \frac{2}{5} \\
F(u) &= \frac{3}{5} u^{5/3} + \frac{2}{5}
\end{align*}
\]

Q. What is \( \int_{-1}^{1} \frac{1}{1+x^2} \, dx \)?

\[
\tan^{-1} x \Big|_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}
\]

Rk. Actually, there are lots of angles \( \theta \) with \( \tan(\theta) = 1 \).

\[
\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{-3\pi}{4}, \ldots
\]

The definition of \( \tan^{-1} \) says always pick a \( \theta \) between \(-\frac{\pi}{2}\) and \(\frac{\pi}{2}\).}

\[
(\text{For } \sin^{-1}, \text{ take } \theta \text{ between } -\frac{\pi}{2} \text{ and } \frac{\pi}{2})
\]

\[
(\text{For } \cos^{-1}, \text{ take } \theta = 0 \text{ and } \pi)
\]
Net change

Suppose we have a function $F(t)$. \( \frac{dF}{dt} = F'(t) \) is the rate of change of $F$.

\[
\int_a^b F'(t) \, dt = F(b) - F(a) = \text{net change of } F(t) \text{ as } t \text{ goes from } a \text{ to } b.
\]

Q. Sand is flowing into a basin at the rate \((10t + 6) \text{ ft}^3/\text{s}\) (\(t\) measured in s).

The basin contains 400 ft$^3$ of sand at time \(t = 0\).

How much does it contain at time \(t = 10\)?

Total sand = initial + \(\int_0^{10} (\text{rate}) \, dt\)

\[
= 400 + \int_0^{10} (10t + 6) \, dt
\]
\[
= 4000 + \left( \frac{10b^2}{2} + 6t \right) \left| _0^6 \right.
\]
\[
= 4000 + (5t^2 + 6t) \left| _0^6 \right.
\]
\[
= 4000 + \left[ (5(10)^2 + 6(10)) - 60 \right]
\]
\[
= 4000 + (5(100) + 60)
\]
\[
= 4000 + 560 + 60
\]
\[
= 4620 F \text{ A>B}
\]

Q. A rechargeable battery is connected to a load that can charge or discharge it. The current flowing into it is
\[
\sin(\pi t) + \frac{1}{2}
\]
(t in days)

The battery starts with 10 units of charge at \( t=0 \).
How much does it have at \( t=6 \)? (after 6 days)

\[
F(t) = \text{charge at time } t
\]
\[
F(6) - F(0) = \int_0^6 \left( \sin(\pi t) + \frac{1}{2} \right) \, dt
\]
\[
= \left. -\cos(\pi t) + \frac{t}{2} \right|_0^6
\]
\[
= \left( -\frac{1}{\pi} + \frac{6}{2} \right) - \left( -\frac{1}{\pi} + 0 \right)
\]
\[
= \left( \frac{1}{\pi} + 3 \right) - \left( \frac{1}{\pi} + 0 \right)
\]
\[
= 3
\]
\[ F(b) - 10 = 3 \]
\[ F(b) = 13 \]

**Total displacement vs total distance**

Recall: if \( s(t) = \text{position (along a line)} \)

\[ s'(t) = v(t) = \text{velocity} \]

\[
\begin{cases}
  v(t) > 0: \text{ moving to the right} \\
  v(t) < 0: \text{ moving to the left}
\end{cases}
\]

Speedometer shows speed: \( \text{speed} = |v(t)| \)

\[
\begin{array}{c}
\text{total displacement} \\
\text{total distance}
\end{array}
\]

\[
\begin{align*}
  s(b) - s(a) &= \int_a^b v(t) \, dt = A_1 - A_2 + A_3 \\
  \int_a^b |v(t)| \, dt &= A_1 + A_2 + A_3
\end{align*}
\]
A particle moves along a line with \( v(t) = t^2 - t - 6 \) m/s \((t \text{ in s})\) from time \( t = 1 \) to \( t = 4 \).

a) What is the total displacement?

b) What is the total distance?

a) displacement = \( \int_{1}^{4} v(t) \, dt = \int_{1}^{4} (t^2 - t - 6) \, dt \)

\[ = \left[ \frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_{1}^{4} \]

\[ = \left( \frac{64}{3} - 8 - 24 \right) - \left( \frac{1}{3} - \frac{1}{2} - 6 \right) \]

\[ = \frac{64}{3} - 8 - 24 - \frac{1}{3} + \frac{1}{2} + 6 \]

\[ = \frac{9}{2} \text{ m} \]

(\( \frac{9}{2} \text{ m to the left} \))

b) total distance = \( \int_{1}^{4} |v(t)| \, dt \)

make a "sign chart" for \( v(t) \):

\[ v(t) = (t-3)(t+2) \]

use that to get \( \int |v(t)| \, dt \)