Coordinates in 3 dimensions (Ch 12.1)

Up to now, we worked in the plane.

Now, work in 3 dimensions:

space is divided into 8 regions ("octants")

positive octant: \[ x > 0 \]
\[ y > 0 \]
\[ z > 0 \]

like a room, origin \((x,y,z) = (0,0,0)\) at corner

x-y plane is floor: \( z = 0 \)
x-z plane is a wall: \( y = 0 \)
y-z plane is a wall: \( x = 0 \)

Any point in 3-dim space can be specified by 3 #'s \((x,y,z)\)

Ex. Draw the locus \( \{ z = 3 \} \) in 3 dim.
(i.e. the set of all points obeying this equation)

It's a plane.
Imposing a linear equation on \((x,y,z)\) usually gives something **2-dimensional**.

**Example**

Draw the locus \(\{y=5\}\) in 3 dim.

Again a **plane**.

**Example**

Draw \(\{x=y^2\}\) in 3 dim.

Get a **plane** lying over the line \(x=y\) in the \(xy\)-plane.

In fact, any eq. of the form

\[Ax + By + Cz + D = 0\]

**gives a plane.**

**Example**

\((x^2 + y^2 = 1\) and \(z=2)\)

→ **circle in the** \(z=2\) **plane**

   center at \((0,0,2)\)

   radius 1

**Example**

\(x^2 + y^2 = 9\)

→ **cylinder around** \(z\)-axis, radius 3
Distance  Given two points  \( A = (x_1, y_1, z_1) \)  \( B = (x_2, y_2, z_2) \)

the distance from \( A \) to \( B \) is  

\[
\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}
\]

\[
\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}
\]

Ex  Distance from \( (1, -1, 4) \) to \( (2, 3, 0) \) is

\[
\sqrt{(1-2)^2 + (-1-3)^2 + (4-0)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}
\]

Why does this formula work?

Just Pythagoras. Then twice!

Ex  What is the locus \( \{ x^2 + y^2 + z^2 = 25 \} \)?
\[ x^2 + y^2 + z^2 \] is the distance from \((0,0,0)\) to \((x,y,z)\). So, we have a sphere centered at \((0,0,0)\) with radius \(5\).

Similarly: \( (x-h)^2 + (y-k)^2 + (z-l)^2 = r^2 \) is sphere centered at \((h,k,l)\) w/ radius \(r\).

**Ex:** What is the distance from \((4,2,3)\) to the y-axis?

The y-axis is given by \(x=0, \ z=0\).

So, any point on the y-axis is \((0, y, 0)\).

Distance from \((4,2,3)\) to \((0, y, 0)\) is \(\sqrt{(4-0)^2 + (2-y)^2 + (3-0)^2}\).

\[ = \sqrt{25 + (2-y)^2} \]

The minimum distance is attained by setting \(y = 2\).

Then, set distance \(= \sqrt{25} = 5\).
Vectors (Ch 12.2)

A vector is a quantity with magnitude and direction.

Picture it as an arrow:

- length and direction are the only information it knows (does not care where it starts)

Ex: If have two points $A, B$:

$\overrightarrow{AB}$

"displacement vector" from $A \rightarrow B$

Adding vectors: Given vectors $\overrightarrow{u}, \overrightarrow{v}$ define a new vector $\overrightarrow{u} + \overrightarrow{v}$

(also called "parallelogram law":

$\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}$)

Scalar multiplication: Given a vector $\overrightarrow{u}$ and a real number $\lambda$, define $\lambda \overrightarrow{u}$ to be a vector which:

- if $\lambda > 0$, points same direction as $\overrightarrow{u}$ and has length $\lambda$ times the length of $\overrightarrow{u}$
- if $\lambda < 0$, points opposite direction as $\overrightarrow{u}$ and has length $|\lambda|$ times the length of $\overrightarrow{u}$

- if $\lambda = 0$, zero vector
If \( x < 0 \), point opposite dir from \( \vec{u} \), length \( |x| \) times the length of \( \vec{u} \).

Subtract: \[ \vec{u} - \vec{v} = \vec{u} + (-1) \cdot \vec{v} \]

\[-\vec{v} = (-1) \cdot \vec{v} \]

How to represent a vector concretely?

Put tail of \( \vec{v} \) at \((0,0,0)\) in \((x,y,z)\) coords.

Call the coords of the point at the tip of \( \vec{v} \) the "components" of \( \vec{v} \). Write \[ \vec{v} = \langle x, y, z \rangle \]

Ex: Say \( \vec{A} = (1,2,-4) \) \( \vec{B} = (3,4,7) \)

What is \( \vec{AB} \)? \[ \langle 2, 2, 11 \rangle \]
Ex: Say \( A = (1, 3, 8) \), \( B = (0, 5, 6) \)

What is \( \overrightarrow{AB} \)? \( \langle -1, 2, -2 \rangle \) \[ \overrightarrow{BA} = \langle 1, -2, 2 \rangle \]

Generally, if \( A = (x_1, y_1, z_1) \), \( B = (x_2, y_2, z_2) \) then \( \overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \)