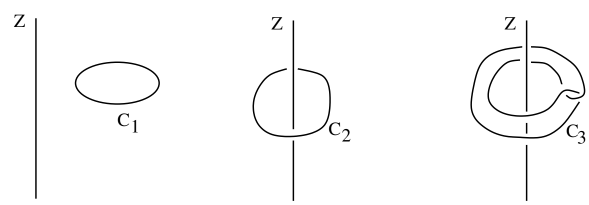
Name:
Date:
Due: Friday, December 7
Homework 10
Do all of the problems.

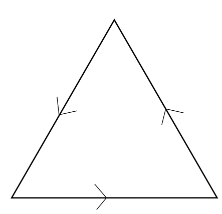
These problems all come from prelim exams. The semester and year are indicated above the question.

2007 Spring
3. Let $Z \subset \mathbb{R}^3$ denote the z-axis, and let C_1, C_2, and C_3 denote the pictured simple closed curves in $\mathbb{R}^3$:



(a) Prove that there is no homeomorphism $f : \mathbb{R}^3 \to \mathbb{R}^3$ sending Z onto inself and C_1 onto C_2.
(b) Prove that there is no homeomorphism $f : \mathbb{R}^3 \to \mathbb{R}^3$ sending Z onto inself and C_1 onto C_3. (Hint: Consider the unversal cover of $\mathbb{R}^3 - Z$).

2006 Fall
A2. Let W be the space obtained from a 2-simplex by identifying its edges as pictured.



(a). Find all covering spaces of W. Draw pictures for each and prove that you have found them all. (hint: π_1)

(b). An edge of the triangle becomes a closed curve in W. Show there is no retraction of W onto this curve.

Fall 2004

1. Let's define the 3-hole connected sum of two closed connected surfaces, M_1^2 M_2^2 as follows: Let D_1, D_2, D_3 be three disjoint disks in M_1^2 and let E_1, E_2, E_3 be three disjoint disks in M_2^2. Then the "3-hole connected sum" of M_1^2 and M_2^2 is the space created by removing the interiors of D_1, D_2, and D_3 from M_1^2, removing the interiors of E_1, E_2, and E_3 from M_2^2, and identifying ∂D_1 with ∂E_1, ∂D_2 with ∂E_2, and ∂D_3 with ∂E_3 via homeomorphisms ("∂" denotes boundary).

(a) Is the 3-hole connected sum well-defined up to homeomorphism? Explain.

(b) Describe all the surfaces that can result from a 3-hole connected sum of a Klein bottle K^2 with a genus 2 "double torus" M^2 . Explain your answers.

(Hint: First consider the two holed connected sum operation.)