Theorem 1. *Any polygonal disk with edges identified in pairs is homeomorphic to a compact, connected, triangulated* 2*-manifold.*

Theorem 2. *Any compact, connected, triangulated* 2*-manifold is homeomorphic to a polygonal disk with edges identified in pairs.*

Exercise 3. *The boundary of a tetrahedron is naturally triangulated with a triangulation* T *consisting of four* 2*-simplexes together with their six edges and four vertices. On the boundary of a tetrahedron locate the first and second derived subdivisions of* T*, the* 1*-skeleton of* T*, the regular neighborhood of the* 1*-skeleton of* T*, the regular neighborhoods of a vertex and an edge of* T*, and the dual* 1*-skeleton of* T*.*

Exercise 4. *On the accompanying pictures of the second derived subdivisions of triangulations of the torus and the Klein bottle, find regular neighborhoods of subsets of the* 1*-skeleton.*

Exercise 5. *Characterize graphs in the* 1*-skeleton of* T *for the triangulations of the sphere, torus, and projective plane whose regular neighborhoods are homeomorphic to a disk.*

Theorem 6.

1. *The bigon with edges identified by* aa^{-1} *is homeomorphic to* $\mathbb{S}^2$*.*
2. *The bigon with edges identified by* bb *is homeomorphic to* $\mathbb{R}\mathrm{P}^2$*.*
3. *The square with edges identified by* $cdc^{-1}d^{-1}$ *is homeomorphic to* $\mathbb{T}^2$*.*

Theorem 7 (connected sum relation)**.** *The gluing of a square given by* $ccdd$ *is homeomorphic to* $\mathbb{R}\mathrm{P}^2\#\mathbb{R}\mathrm{P}^2$ *and the gluing of an octagon given by* $aba^{-1}b^{-1}cdc^{-1}d^{-1}$ *is homeomorphic to* $\mathbb{T}^2\#\mathbb{T}^2$*.*

Question 8. *Generalize the above to the connected sum of any two surfaces.*

Theorem 9. *Let* $Abb^{-1}C$ *be a string of* $2n$ *letters where each letter occurs twice, with or without a superscript (so* A *and* C *should each be construed as being comprised of many letters). Then the* 2*-manifold obtained by identifying a* $2n$*-gon following the gluing* $Abb^{-1}C$ *is homeomorphic to the* 2*-manifold which is obtained by identifying a* $(2n-2)$*-gon following the gluing given by* AC*.*

Theorem 10. *Suppose a* 2*-manifold* M^2 *is represented by a* $2n$*-gon with edges identified in pairs. Then a homeomorphic* 2*-manifold can be represented by a* $2k$*-gon with edges identified in pairs where all the vertices are in the same equivalence class, that is, all the vertices are identified to each other.*

Theorem 11. *Suppose a* 2*-manifold* M^2 *is represented by a* $2n$*-gon with edges identified in pairs. Then a homeomorphic* 2*-manifold can be represented by a* $2k$*-gon with edges identified in pairs where all the vertices are identified and every pair of edges with the same orientation are consecutive.*

Theorem 12. *Suppose a* 2*-manifold* M^2 *is represented by a* $2n$*-gon with edges identified in pairs. Then a homeomorphic* 2*-manifold can be represented by a* $2k$*-gon with edges identified in pairs where all the vertices are identified, every pair of edges with the same orientation are consecutive, and all other edges are grouped in disjoint sets of two intertwined pairs following the pattern* $aba^{-1}b^{-1}$.

Theorem 13. *The* 2*-manifold represented by* $aba^{-1}b^{-1}cc$ *is homeomorphic to the* 2*-manifold represented by* $ddeeff$.

Question 14. *Re-state the above theorem in terms of connected sum.*

Theorem 15. *Any compact, connected, triangulated* 2*-manifold is homeomorphic to a* $2n$*-gon with edges identified in pairs as specified in one of the three following ways:* aa^{-1}, *or* $a_0a_0a_1a_1 \ldots a_na_n$ *(where* $n \geq 0$*) or* $a_0a_1a_0^{-1}a_1^{-1} \ldots a_{n-1}a_na_{n-1}^{-1}a_n^{-1}$ *(where* $n \geq 1$ *is odd).*

Theorem 16 (Classification of compact, connected 2-manifolds)**.** *Any connected, compact, triangulated 2-manifold is homeomorphic to the 2-sphere* $\mathbb{S}^2$*, a connected sum of tori, or a connected sum of projective planes.*