Name:
Date:
Due: 10/22/07
Homework 6 (Chapter 3 3.52-3.66)
Do all of the problems.

3.6 3-manifolds

3.6.1 Lens spaces

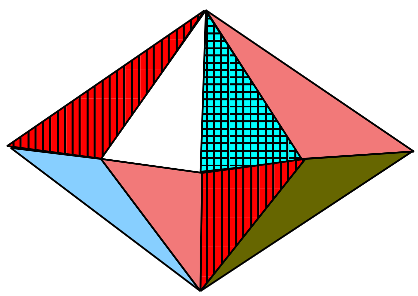

Figure 3.8: Lens space as a quotient of a lens

Exercise 1. *Show that isotopies form an equivalence relation on the set of all embeddings of X into Y.*

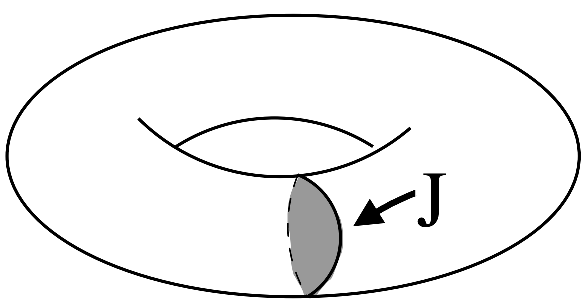

Figure 3.9: Solid torus with meridian

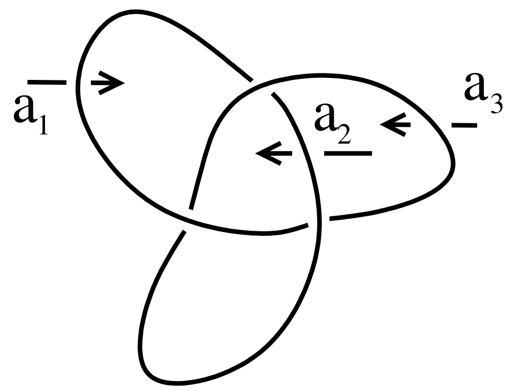

Figure 3.11: The arrows for the arcs of a trefoil knot

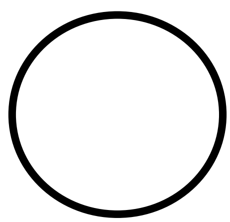

Figure 3.13: The unknot

Lemma 55. *Let $\{[\mu], [\lambda]\}$ be a basis for $\pi_1(Bd(\mathbb{D}^2 \times \mathbb{S}^1)) \cong \mathbb{Z} \times \mathbb{Z}$. Then $p[\mu] + q[\lambda]$ has a simple closed curve representative if and only if p and q are relatively prime.*

Exercise 56. *Use Van Kampen's Theorem to explicitly calculate a group presentation of $\pi_1(L(p, q))$.*

3.6.2 Knots in $\mathbb{S}^3$

Lemma 57. *Every loop in M_K is homotopic in M_K to a product of a_i's. In other words, the loops $\{a_i\}$ generate $\pi_1(M_K)$.*

Exercise 60. *Find the fundamental group of the complement of the unknot (See Figure 3.13).*

Exercise 61. *Find the fundamental group of the complement of the trefoil knot.*

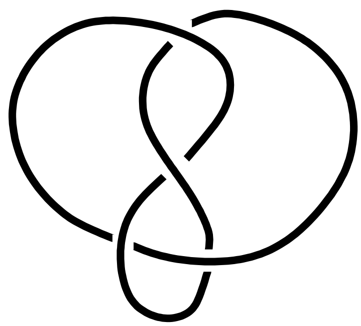

Figure 3.14: The figure-8 knot

Exercise 62. *Find the fundamental group of the complement of the figure-8 knot (See Figure 3.14).*

3.7 Homotopy equivalence of spaces

Theorem 64. *If $X \sim Y$ then $\pi_1(X) \cong \pi_1(Y)$.*