Name:
Date:
Due: 11/2/07
Homework 7 (Chapter 3 3.67-3.105)
Do all of the problems.

3.9 Covering spaces

Theorem 3.68. *Let* $(\widetilde{X}, p)$ *be a covering space of* X*. If* x*,* $y \in X$*, then* $|p^{-1}(x)| = |p^{-1}(y)|$*.*

Exercise 3.69.

> 3. *Describe all non-homeomorphic 3-fold covers of the wedge of two circles.*

Exercise 3.72.

1. *Describe all non-homeomorphic 3-fold covers of the Klein bottle.*
2. *Describe all non-homeomorphic 2-fold covers of* $\mathbb{T}^2 \# \mathbb{T}^2$*.*
3. *Describe all non-homeomorphic 3-fold covers of* $\mathbb{T}^2 \# \mathbb{T}^2 \# \mathbb{T}^2$*.*
4. *Describe all non-homeomorphic 3-fold covers of* $\mathbb{R}\mathrm{P}^2$*.*

Theorem 3.74. *If* $(\widetilde{X}, p)$ *is a cover of* X*,* Y *is connected, and* f*,* $g : Y \to \widetilde{X}$ *are continuous functions such that* $p \circ f = p \circ g$*, then* $\{y \mid f(y) = g(y)\}$ *is empty or all of* Y*.*

Theorem 3.75. *Let* $(\widetilde{X}, p)$ *be a cover of* X *and let* f *be a path in* X*. Then for each* $x_0 \in \widetilde{X}$ *such that* $p(x_0) = f(0)$*, there exists a unique lift* $\widetilde{f}$ *of* f *satisfying* $\widetilde{f}(0) = x_0$*.*

Theorem 3.78. *If* $(\widetilde{X}, p)$ *is a cover of* X*, then* p_* *is a monomorphism (*i.e.*, 1–1 or injective) from* $\pi_1(\widetilde{X})$ *into* $\pi_1(X)$*.*

Theorem 3.79. *Let* $(\widetilde{X}, p)$ *be a cover of* X*,* α *a loop in* X*, and* $\widetilde{x}_0 \in \widetilde{X}$ *such that* $p(\widetilde{x}_0) = \alpha(0)$*. Then* α *lifts to a loop based at* $\widetilde{x}_0$ *if and only if* $[\alpha] \in p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$*.*

Theorem 3.82. *Let* $(\widetilde{X}, p)$ *be a covering space of* X*. Choose* $x \in X$*, then* $|p^{-1}(x)| = [\pi_1(X) : p_*(\pi_1(\widetilde{X}))]$*.*

Exercise 3.93.

1. *Describe all regular 3-fold covering spaces of a figure eight.*
2. *Describe all irregular 3-fold covering spaces of a figure eight.*
3. *Describe all regular 4-fold covering spaces of a figure eight.*
4. *Describe all irregular 4-fold covering spaces of a figure eight.*
5. *Describe all regular 3-fold covering spaces of a wedge of 3 circles.*
6. *Describe all regular 4-fold covering spaces of a wedge of 3 circles.*

Theorem 3.94. *Let $(\widetilde{X}, p)$ be a regular covering space of X. Then $\mathcal{C}(\widetilde{X}, p) \cong \pi_1(X)/p_*(\pi_1(\widetilde{X}))$. In particular, $\mathcal{C}(\widetilde{X}, p) \cong \pi_1(X)$ if $\widetilde{X}$ is simply connected.*

3.10 Theorems about groups

Corollary 3.101. *A subgroup H of a free group F_n is always a free group.*