

Name:

Date:

Due: Monday, December 3rd

Homework 9

Do all of the problems.

These problems all come from prelim exams. The semester and year are indicated above the question.

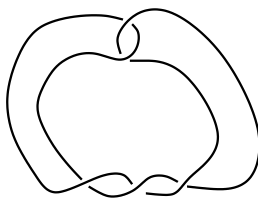
2005 Spring

1) Let X be the wedge of the three spaces RP^2 , S^2 and S^3 and let \tilde{X} be the universal cover of X . Compute the homology groups $H_i(\tilde{X})$ for all i .

2. a) Let S be a compact solid torus. Compute all the homology groups of S and compute all of the homology groups of ∂S .

b) Let T be the (long and thin) knotted solid torus in R^3 shown below, and let S^3 be the one-point compactification of R^3 . Compute all the homology groups of $\overline{S^3 - T}$ (the closure of the complement of T in S^3).

(Hint: First think about about $S^3 - K$ where K is the unknot.)



August 2005

1. (a) Describe a CW decomposition for the Klein Bottle.

(b) Use that CW decomposition to compute the homology groups in all dimensions of the Klein Bottle.

(c) Now consider the Klein Bottle as the union of two Moebius bands with the boundary S^1 of one Moebius band identified with the boundary S^1 of the other Moebius band. Using this decomposition compute the fundamental group of the Klein Bottle using Van Kampen's Theorem.

(d) Use the same decomposition to compute all the homology groups of the Klein Bottle using the Mayer-Vietoris Theorem.

Spring 2003

2. Let T_1 and T_2 be tori and J_1 and J_2 be homotopically trivial simple closed curves on T_1 and T_2 respectively. Let X be the quotient space obtained by identifying J_1 and J_2 by a homeomorphism.

- (a) Use Van Kampens Theorem to compute the fundamental group of X .
- (b) Use the Mayer-Vietoris Theorem to compute all homology groups of X .
- (c) Is X homotopy equivalent to $T_1 \times T_2$?

Fall 2003

3.(a) A 2-disc D^2 is attached to a 2-torus T^2 by a map $f : S^1 = \partial D^2 \rightarrow T^2$. If $X = T^2 \cup_f D^2$ what are the possible homology groups $H_i(X; Z)$? (b) A 3-disc D^3 is attached to a 2-torus T^2 by a map $g : S^2 = \partial D^3 \rightarrow T^2$. If $Y = T^2 \cup_g D^3$ what are the possible homology groups $H_i(Y; Z)$?