

$$\frac{1}{2\beta_1} = \frac{1}{2}$$

Problem 5. Show that:

a)  $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin(\alpha) \cos(\beta)$ .

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin(\alpha) \cos \beta + \sin \beta \cos \alpha + \sin(\alpha) \cos \beta - \sin \beta \cos \alpha$$

Using

$$\sin(\alpha + \beta) = \sin(\alpha) \cos \beta + \sin(\beta) \cos(\alpha)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \cos \beta - \sin(\beta) \cos(\alpha)$$

$$\Rightarrow = 2 \sin(\alpha) \cos(\beta) \quad \checkmark$$

b)  $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$

$$\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\frac{\sin \theta}{1 + \sin \theta} \cos \theta}{1 + \sin \theta} = \frac{\sin \theta + \frac{\sin^2 \theta + \cos^2 \theta}{1 + \sin \theta}}{\cos \theta}$$

Using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$= \frac{\sin \theta + 1}{\cos \theta (1 + \sin \theta)} = \frac{1}{\cos \theta} = \sec \theta$$

Using  $\sin^2 \theta + \cos^2 \theta = 1$

Using  $\sec \theta = \frac{1}{\cos \theta}$

$$1 + \sin \theta = \sin \theta + 1$$