

Problem 6. Show that $\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

$$\frac{1-\sin\theta}{1-\sin\theta} \cdot \frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} \cdot \frac{\cos\theta}{\cos\theta}$$

$$\frac{1-\sin\theta}{1-\sin\theta} \cdot \frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} = \frac{1-2\sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)} + \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{1-2\sin\theta + 1}{\cos\theta(1-\sin\theta)} = \frac{2-2\sin\theta}{\cos\theta(1-\sin\theta)}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta(1-\sin\theta)} = 1$$

$$= \frac{2 \cdot 1}{\cos\theta} = 2\sec\theta$$

$$\sec\theta = \frac{1}{\cos\theta}$$