

Problem 8. Given $\sin \theta = \frac{3}{7}$ and $-\pi/2 < \theta < \pi/2$ find:

a) $\sin(2\theta)$

b) $\cos(2\theta)$

c) $\tan(2\theta)$

d) $\sin(\frac{\theta}{2})$

e) $\cos(\frac{\theta}{2})$

f) $\tan(\frac{\theta}{2})$

$$\sin(2\theta) = 2\sin\theta \cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$a) \sin\theta = \frac{3}{7} = \frac{y}{r}$$

$$y=3 \quad r=7$$

$$x^2 + y^2 = r^2$$

$$x^2 + 9 = 49$$

$$x^2 = 40$$

$$x = \sqrt{40} = 2\sqrt{10}$$

$$\cos\theta = \frac{2\sqrt{10}}{7}$$

$$\tan\theta = \frac{3}{\frac{2\sqrt{10}}{7}} = \frac{3\sqrt{10}}{2}$$

$$\sin 2\theta = 2\left(\frac{3}{7}\right)\left(\frac{2\sqrt{10}}{7}\right) = \frac{12\sqrt{10}}{49}$$

$$\cos(2\theta) = \left(\frac{2\sqrt{10}}{7}\right)^2 - \left(\frac{3}{7}\right)^2 = \frac{40}{49} - \frac{9}{49} = \frac{31}{49}$$

$$\tan(2\theta) = \frac{2 \cdot \frac{3\sqrt{10}}{7}}{1 - \left(\frac{3\sqrt{10}}{7}\right)^2} = \frac{\frac{6\sqrt{10}}{7}}{1 - \frac{30}{49}} = \frac{\frac{6\sqrt{10}}{7}}{\frac{19}{49}} = \frac{6\sqrt{10}}{7} \cdot \frac{49}{19} = \frac{6\sqrt{10}}{19}$$

$$\frac{3\sqrt{10}}{20} = \frac{31}{40}$$

$$\frac{3\sqrt{40}}{20} = \frac{31}{40}$$

$$\frac{40}{31} = \frac{6\sqrt{40}}{31}$$