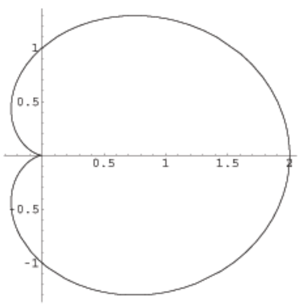
Problem 1

a) Make a table of values for θ and $r(\theta) = 1 + \cos\theta$

θ	$r(\theta)$
0	2
$\pi/4$	$1 + 1/\sqrt{2}$
$\pi/2$	1
$\pi/4$	$1 - 1/\sqrt{2}$
π	0
$5\pi/4$	$1 - 1/\sqrt{2}$
$3\pi/2$	1
$7\pi/4$	$1 + 1/\sqrt{2}$
2π	2

b) Graph $r(\theta)$



c) What happens if we graph $r(\theta) = 1 + \cos 2\theta$?

θ	$r(\theta)$
0	2
$\pi/8$	$1 + 1/\sqrt{2}$ $\pi/4$
1	
$\pi/2$	0
$3\pi/4$	1
$7\pi/8$	$1 + 1/\sqrt{2}$
$5\pi/4$	1
$3\pi/2$	0
$7\pi/4$	$1 + 1/\sqrt{2}$
2π	2

The graph changes dramatically. For one thing, r=0 at $\pi/2$ and $3\pi/2$. Actually, you can see this on the graph it touches the origin in twice.

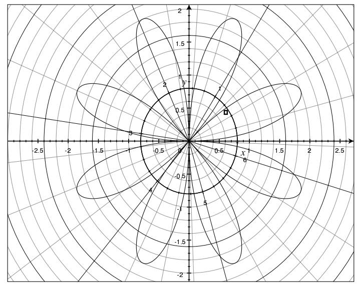
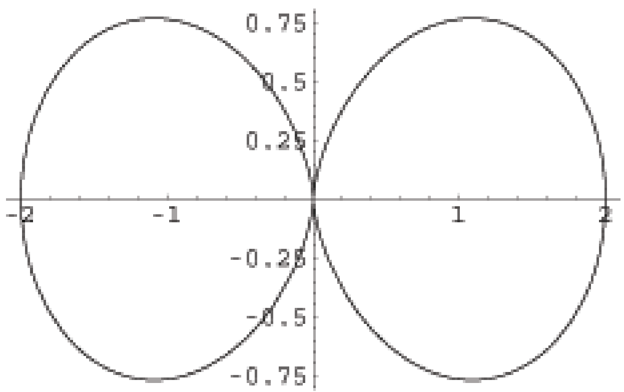


Figure 1: The graph of $r(\theta) = 2\sin 4\theta$ and the circle of radius 1



Problem 2:

a) Look at the curve $r(\theta) = 2\sin 4\theta$. What values can r take on? What values of θ make r=0?

Since $-1 \leq \sin 4\theta \leq 1$, $-2 \leq r(\theta) \leq 2$. If $\sin 4\theta = 0$, then $r = 0$. Notice this happens when $4\theta = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi$ and 8π or when $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$ and 2π.

b) Graph $r(\theta)$. What symmetries does the graph have? How can these symmetries be useful?

The graph has an 8 fold rotational symmetry. This means that we really only need to draw one of the pedals and then rotate it by an angle of $\pi/4$ in order to make the whole graph.

c) When do $r(\theta)$ and the circle of radius 1 have common points?

We want to know when $r(\theta) = \pm 1$, so $2\sin 4\theta = \pm 1$. Here $\sin 4\theta = \pm 1/2$, so $4\theta = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6, 13\pi/6,$

So, if $\theta = \pi/24, 5\pi/24, 7\pi/24, 11\pi/24, 13\pi/24, 17\pi/24, 19\pi/24, 23\pi/24, 25\pi/24, 29\pi/24, 31\pi/24, 35\pi/24, 37\pi/24$ the graphs have common points.

(Again, the symmetry is useful here.)

Problem 3:

a) Look at $x = \frac{1}{\sin t}$, $y = \frac{1}{\sin^2 t}$ where $0 < t \leq \pi/2$. What values can x,y take on?

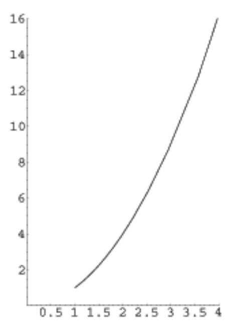
If $0 < t \leq \pi/2$, $0 < \sin t \leq 1$. So, $\frac{1}{\sin t} \geq 1$ and $x \geq 1$. $y = \frac{1}{\sin^2 t} \geq 1$

b) Make a table of values for t, x, and y.

t	x	y
0	DNE	DNE
$\pi/6$	2	4
$\pi/4$	$\sqrt{2}$	2
$\pi/3$	$2/\sqrt{3}$	$4/3$
$\pi/2$	1	1

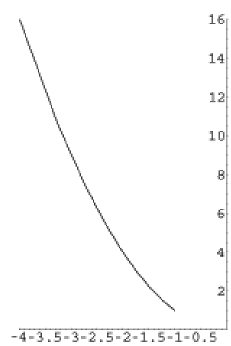
Do you notice relationship between x and y? Use this information to graph the curve.

Looking at x and y, we see $y = x^2$. So, the points we plot should be a subset of $y = x^2$.



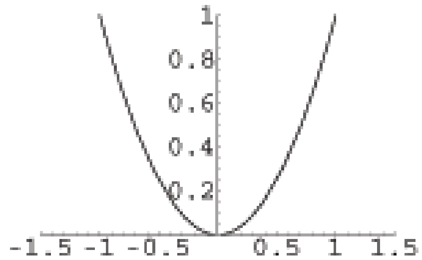
Notice, if we let $0 < t < \pi$, the graph will look the same, but will be traced over twice.

c) How would things change if $\pi < t < 2\pi$?



If we let $\pi < t < 2\pi$, then x is negative. In fact, $x \leq -1$ and $y \geq 1$.

d) How would things change if $x = \sin t$ and $y = \sin^2 t$?



So, $-1 \leq \sin t \leq 1$, $-1 \leq x \leq 1$, and $0 \leq y \leq 1$. We still have that $y = x^2$. It's just going to be plotting points on a different part of the graph.