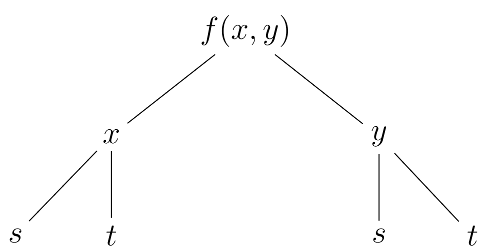
1) Find all first partials of the following.

a) $f(x, y) = y^2 \sin(x - y)$

$f_x = y^2 \cos(x - y)(1) = y^2 \cos(x - y)$
For f_y, we need to use the product rule. $f_y = 2y \sin(x - y) + y^2 \cos(x - y)(-1) = 2y \sin(x - y) - y^2 \cos(x - y)$
b) $f(x, y) = \frac{x}{x^2+y^2}$

For f_x, we need to use the quotient rule. $f_x = \frac{(x^2+y^2)(1)-x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{x^2+y^2}$
$f_y = x(x^2 + y^2)^{-1} = (-1)(x)(x^2 + y^2)^{-2}(2y) = \frac{-2xy}{(x^2+y^2)^{-2}}$
2. Let $f(x, y) = \sqrt{x^2 - y^2}$. Let $x = s - t$, $y = s + t$.
a) Draw a tree diagram for the function.



b) State the chain rule for $\frac{\partial f}{\partial s}$.

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial x} \cdot \frac{\partial y}{\partial s}$$

Notice that here we need ∂ not d
c) Use the chain rule to compute $\frac{\partial f}{\partial s}$.

From b), we have:
$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial x} \cdot \frac{\partial y}{\partial s}$$
Now it's just a matter of plugging into this equation. So let's do it piece by piece.
$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2)^{\frac{1}{2}} = \frac{1}{2(x^2-y^2)^{\frac{1}{2}}} \cdot (2x) = \frac{x}{(x^2-y^2)^{\frac{1}{2}}}$
$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2)^{\frac{1}{2}} = \frac{1}{2(x^2-y^2)^{\frac{1}{2}}} \cdot (-2y) = \frac{-y}{(x^2-y^2)^{\frac{1}{2}}}$

$\frac{\partial x}{\partial s} = 1$
$\frac{\partial y}{\partial s} = 1$
Now, we can plug into the equation we got from b).

$$\frac{\partial f}{\partial s} = \frac{x}{(x^2-y^2)^{\frac{1}{2}}} + \frac{-y}{(x^2-y^2)^{\frac{1}{2}}} = \frac{x-y}{(x^2-y^2)^{\frac{1}{2}}}$$

If you like you can plug in $x = s - t$, $y = s + t$.
$x^2 - y^2 = (s-t)^2 + (s+t)^2 = s^2 - 2st + t^2 - (s^2 + 2st + t^2) = -4st$
$x - y = s - t - s - t = -2t$
So,

$$\frac{\partial f}{\partial s} = \frac{-2t}{(-4st)^{\frac{1}{2}}} = \frac{-t}{(-st)^{\frac{1}{2}}}$$

3. Let $z = f(x,y) = y^2 - x^2$
a) Sketch the surface.
b) Sketch the trace of the $z = f(x,0)$ in the xz plane amd pm the surface.
c) Sketch the trace of the $z = f(1,y)$ in the yz plane amd pm the surface.

See http://www.ma.utexas.edu/ nhoffman/Spring2007/FQ9Question3.pdf for a write up of a), b) and c).
d) Compute $\frac{\partial f}{\partial x}(.5,0)$ and explain its geometric meaning.
First we need to compute $\frac{\partial f}{\partial x}$.
$\frac{\partial f}{\partial x} = -2x$
$\frac{\partial f}{\partial x}(.5,0) = -2(.5) = -1$
The fact that $\frac{\partial f}{\partial x}$ is negative means the function is decreasing in x at $(.5,0)$. Also for part 3b) we would see a tangent line at x=.5, z=-.25 with slope -1.
e) Compute $\frac{\partial f}{\partial y}(1,0)$ and explain its geometric meaning.
$\frac{\partial f}{\partial y} = 2y$
$\frac{\partial f}{\partial y}(1,0) = 0$
The fact that $\frac{\partial f}{\partial y}(1,0)$ is zero means the function is neither increasing or decreasing in y. In fact when we look at the tangent line to the trace of $z = y^2 - 1$ (Look at 3c)) we see that the tangent line to the curve is horizontal. This along with the fact that $\frac{\partial f}{\partial y} < 0$ for $y < 0$ and $\frac{\partial f}{\partial y} > 0$ for $y > 0$ indicates that at (1,0) the function goes from being decreasing in y to begin increasing in y.