Practice Midterm 2 for Math 341

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1. Let A and B be defined as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 6 & 11 \end{bmatrix}$$

   (a) Demonstrate that A and B row equivalent by providing a sequence of row operations leading from A to B.

   (b) Check whether $\vec{x} = [1, 2, 3]$ is in the row space of A, and if it is, write it as a linear combination of the rows of A.

   (c) Is $\vec{x}$ in the row space of B? (You shouldn't need many calculations here...)

2. Prove that $R(AB) = R(A)B$ if R is the row operation Row $1 \to 2\times$Row 1.

   Hint: Show that the (i, j) entry of $R(AB)$ is equal to the (i, j) entry of $R(A)B$. You'll have to consider $i = 1$ and $i \neq 1$ separately!

3. Let A be defined as follows:

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 5 & 3 & 11 \\ -2 & 1 & 0 \end{bmatrix}$$

   In that case (you do not need to check this!), the row reduced echelon form of A is

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

   (a) What is the rank of A?

   (b) Is A singular or nonsingular?

   (c) Check that if $\vec{b} = [2, 3, 1]^T$, then $\vec{x} = [0, 1, 0]^T$ solves the system $A\vec{x} = \vec{b}$.

   (d) Using the information from parts (a), (b), and (c), without doing any calculations, how many solutions does $A\vec{x} = \vec{b}$ have? (Here, $\vec{b}$ is defined as in part (c).)

4. Prove that if A is an $n \times n$ diagonal matrix whose row-reduced echelon form is I_n, then none of the diagonal entries of A are 0.

5. Let A be defined as below:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

   (a) Calculate A^{-1} if A is nonsingular, or prove that it is singular.

   (b) Calculate $|A|$ by using row or column expansion.

   (c) Calculate $|A|$ using row reduction (feel free to reuse your work from part (a) for this!)

6. Let A and B satisfy the following:

$$A = \begin{bmatrix} 0 & ? & ? \\ 1 & ? & ? \\ -1 & ? & ? \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

   That is, we know some entries of A and B but not others.

   (a) Prove that A and B are not inverses of each other.

   (b) Show that $\vec{x} = [1, 2, 1]$ is not in the set $\{\vec{x} \mid \vec{x}A = c[0, 1, 1], c \text{ in } \mathbb{R}\}$.
   Note: Pay attention to the order of multiplication in the definition of that set!!

7. Let A be the matrix defined as

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

   (a) What is the characteristic polynomial $p_A(x)$ of A?

   (b) What are the eigenvalues of A?

   (c) Pick an eigenvalue of A, and write down the fundamental eigenvectors for that eigenvalue.

8. Prove that if

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

   then

$$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$$

   for all positive integers n.