Homework 10

Section 3.4:

12. $f(t) = \sin(e^t) + e^{\sin t} \quad \Rightarrow \quad f'(t) = \cos(e^t) \cdot e^t + e^{\sin t} \cdot \cos t = e^t \cos(e^t) + e^{\sin t} \cos t$

16. $y = e^{-2t} \cos 4t \quad \Rightarrow \quad y' = e^{-2t}(-\sin 4t \cdot 4) + \cos 4t[e^{-2t}(-2)] = -2e^{-2t}(2\sin 4t + \cos 4t)$

30. $F(v) = \left(\dfrac{v}{v^3+1}\right)^6 \quad \Rightarrow$

$$F'(v) = 6\left(\frac{v}{v^3+1}\right)^5 \frac{(v^3+1)(1) - v(3v^2)}{(v^3+1)^2} = \frac{6v^5(v^3+1-3v^3)}{(v^3+1)^5(v^3+1)^2} = \frac{6v^5(1-2v^3)}{(v^3+1)^7}$$

40. $y = \sin(\sin(\sin x)) \quad \Rightarrow \quad y' = \cos(\sin(\sin x)) \dfrac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\, \cos(\sin x)\, \cos x$

50. $y = e^{e^x} \quad \Rightarrow \quad y' = e^{e^x} \cdot (e^x)' = e^{e^x} \cdot e^x \quad \Rightarrow$

$$y'' = e^{e^x} \cdot (e^x)' + e^x \cdot \left(e^{e^x}\right)' = e^{e^x} \cdot e^x + e^x \cdot e^{e^x} \cdot e^x = e^{e^x} \cdot e^x(1+e^x) \quad \text{or} \quad e^{e^x+x}(1+e^x)$$

54. $y = \sin x + \sin^2 x \quad \Rightarrow \quad y' = \cos x + 2\sin x \, \cos x.$

At $(0,0)$, $y' = 1$, and an equation of the tangent line is $y - 0 = 1(x-0)$, or $y = x$.

60. $f(x) = \sin 2x - 2\sin x \quad \Rightarrow \quad f'(x) = 2\cos 2x - 2\cos x = 4\cos^2 x - 2\cos x - 2$, and $4\cos^2 x - 2\cos x - 2 = 0 \quad \Leftrightarrow$

$(\cos x - 1)(4\cos x + 2) = 0 \quad \Leftrightarrow \quad \cos x = 1$ or $\cos x = -\frac{1}{2}$. So $x = 2n\pi$ or $(2n+1)\pi \pm \frac{\pi}{3}$, n any integer.

66. (a) $h(x) = f(f(x)) \quad \Rightarrow \quad h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \quad \Rightarrow \quad g'(x) = f'(x^2) \cdot \dfrac{d}{dx}(x^2) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(2) = 8$.