Homework 12

Section 3.5:

2. (a) $\frac{d}{dx}(2x^2 + x + xy) = \frac{d}{dx}(1) \Rightarrow 4x + 1 + xy' + y \cdot 1 = 0 \Rightarrow xy' = -4x - y - 1 \Rightarrow y' = -\frac{4x + y + 1}{x}$

(b) $2x^2 + x + xy = 1 \Rightarrow xy = 1 - 2x^2 - x \Rightarrow y = \frac{1}{x} - 2x - 1$, so $y' = -\frac{1}{x^2} - 2$

(c) From part (a),

$$y' = -\frac{4x + y + 1}{x} = -4 - \frac{1}{x}y - \frac{1}{x} = -4 - \frac{1}{x}\left(\frac{1}{x} - 2x - 1 - \frac{1}{x}\right) = -4 - \frac{1}{x^2} + 2 + \frac{1}{x} - \frac{1}{x} = -\frac{1}{x^2} - 2, \text{ which}$$

agrees with part (b).

20. $\tan(x - y) = \frac{y}{1 + x^2} \Rightarrow (1 + x^2)\tan(x - y) = y \Rightarrow (1 + x^2)\sec^2(x - y) \cdot (1 - y') + \tan(x - y) \cdot 2x = y' \Rightarrow$

$(1 + x^2)\sec^2(x - y) - (1 + x^2)\sec^2(x - y) \cdot y' + 2x\tan(x - y) = y' \Rightarrow$

$(1 + x^2)\sec^2(x - y) + 2x\tan(x - y) = [1 + (1 + x^2)\sec^2(x - y)] \cdot y' \Rightarrow$

$$y' = \frac{(1 + x^2)\sec^2(x - y) + 2x\tan(x - y)}{1 + (1 + x^2)\sec^2(x - y)}$$

28. $x^2 + 2xy - y^2 + x = 2 \Rightarrow 2x + 2(xy' + y \cdot 1) - 2yy' + 1 = 0 \Rightarrow 2xy' - 2yy' = -2x - 2y - 1 \Rightarrow$

$y'(2x - 2y) = -2x - 2y - 1 \Rightarrow y' = \frac{-2x - 2y - 1}{2x - 2y}$. When $x = 1$ and $y = 2$, we have

$y' = \frac{-2 - 4 - 1}{2 - 4} = \frac{-7}{-2} = \frac{7}{2}$, so an equation of the tangent line is $y - 2 = \frac{7}{2}(x - 1)$ or $y = \frac{7}{2}x - \frac{3}{2}$.

50. $y = \tan^{-1}(x^2) \Rightarrow y' = \frac{1}{1 + (x^2)^2} \cdot \frac{d}{dx}(x^2) = \frac{1}{1 + x^4} \cdot 2x = \frac{2x}{1 + x^4}$

58. $y = \cos^{-1}(\sin^{-1} t) \Rightarrow y' = -\frac{1}{\sqrt{1 - (\sin^{-1} t)^2}} \cdot \frac{d}{dt}\sin^{-1} t = -\frac{1}{\sqrt{1 - (\sin^{-1} t)^2}} \cdot \frac{1}{\sqrt{1 - t^2}}$

Section 3.6:

8. $f(x) = \log_5(xe^x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{xe^x \ln 5}\dfrac{d}{dx}(xe^x) = \dfrac{1}{xe^x \ln 5}(xe^x + e^x \cdot 1) = \dfrac{e^x(x+1)}{xe^x \ln 5} = \dfrac{x+1}{x \ln 5}$

Another solution: We can change the form of the function by first using logarithm properties.

$f(x) = \log_5(xe^x) = \log_5 x + \log_5 e^x \quad \Rightarrow \quad f'(x) = \dfrac{1}{x \ln 5} + \dfrac{1}{e^x \ln 5} \cdot e^x = \dfrac{1}{x \ln 5} + \dfrac{1}{\ln 5}$ or $\dfrac{1+x}{x \ln 5}$

14. $g(r) = r^2 \ln(2r+1) \quad \Rightarrow \quad g'(r) = r^2 \cdot \dfrac{1}{2r+1} \cdot 2 + \ln(2r+1) \cdot 2r = \dfrac{2r^2}{2r+1} + 2r\ln(2r+1)$

32. $f(x) = \ln(1+e^{2x}) \quad \Rightarrow \quad f'(x) = \dfrac{1}{1+e^{2x}}(2e^{2x}) = \dfrac{2e^{2x}}{1+e^{2x}}$, so $f'(0) = \dfrac{2e^0}{1+e^0} = \dfrac{2(1)}{1+1} = 1$.

44. $y = x^{\cos x} \quad \Rightarrow \quad \ln y = \ln x^{\cos x} \quad \Rightarrow \quad \ln y = \cos x \ln x \quad \Rightarrow \quad \dfrac{1}{y}y' = \cos x \cdot \dfrac{1}{x} + \ln x \cdot (-\sin x) \quad \Rightarrow$

$y' = y\left(\dfrac{\cos x}{x} - \ln x \sin x\right) \quad \Rightarrow \quad y' = x^{\cos x}\left(\dfrac{\cos x}{x} - \ln x \sin x\right)$

48. $y = (\sin x)^{\ln x} \quad \Rightarrow \quad \ln y = \ln(\sin x)^{\ln x} \quad \Rightarrow \quad \ln y = \ln x \cdot \ln \sin x \quad \Rightarrow \quad \dfrac{1}{y}y' = \ln x \cdot \dfrac{1}{\sin x} \cdot \cos x + \ln \sin x \cdot \dfrac{1}{x} \quad \Rightarrow$

$y' = y\left(\ln x \cdot \dfrac{\cos x}{\sin x} + \dfrac{\ln \sin x}{x}\right) \quad \Rightarrow \quad y' = (\sin x)^{\ln x}\left(\ln x \cot x + \dfrac{\ln \sin x}{x}\right)$