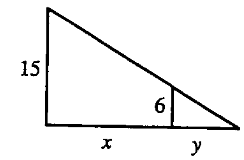
Homework 14

Section 3.9:

13. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\frac{d}{dt}(x+y)$ when $x = 40$ ft.

(c)



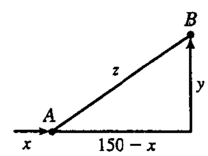
(d) By similar triangles, $\frac{15}{6} = \frac{x+y}{y} \;\Rightarrow\; 15y = 6x + 6y \;\Rightarrow\; 9y = 6x \;\Rightarrow\; y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x+y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{5}{3}\frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

14. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)

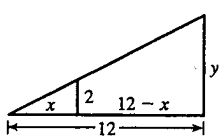


(d) $z^2 = (150-x)^2 + y^2 \;\Rightarrow\; 2z\frac{dz}{dt} = 2(150-x)\left(-\frac{dx}{dt}\right) + 2y\frac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \;\Rightarrow\; z = \sqrt{(150-140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\frac{dz}{dt} = \frac{1}{z}\left[(x-150)\frac{dx}{dt} + y\frac{dy}{dt}\right] = \frac{-10(35)+100(25)}{\sqrt{10{,}100}} = \frac{215}{\sqrt{101}} \approx 21.4$ km/h.

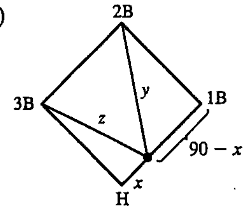
16.



We are given that $\frac{dx}{dt} = 1.6$ m/s. By similar triangles, $\frac{y}{12} = \frac{2}{x} \;\Rightarrow\; y = \frac{24}{x} \;\Rightarrow$ $\frac{dy}{dt} = -\frac{24}{x^2}\frac{dx}{dt} = -\frac{24}{x^2}(1.6)$. When $x = 8$, $\frac{dy}{dt} = -\frac{24(1.6)}{64} = -0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s.

18. We are given that $\frac{dx}{dt} = 24$ ft/s.

(a)



$y^2 = (90-x)^2 + 90^2 \;\Rightarrow\; 2y\frac{dy}{dt} = 2(90-x)\left(-\frac{dx}{dt}\right)$. When $x = 45$, $y = \sqrt{45^2 + 90^2} = 45\sqrt{5}$, so $\frac{dy}{dt} = \frac{90-x}{y}\left(-\frac{dx}{dt}\right) = \frac{45}{45\sqrt{5}}(-24) = -\frac{24}{\sqrt{5}}$, so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$z^2 = x^2 + 90^2 \;\Rightarrow\; 2z\frac{dz}{dt} = 2x\frac{dx}{dt}$. When $x = 45$, $z = 45\sqrt{5}$, so $\frac{dz}{dt} = \frac{45}{45\sqrt{5}}(24) = \frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

22. The distance z of the particle to the origin is given by $z = \sqrt{x^2 + y^2}$, so $z^2 = x^2 + [2\sin(\pi x/2)]^2 \Rightarrow$

$$2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt} + 4\cdot 2\sin\!\left(\frac{\pi}{2}x\right)\cos\!\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}\frac{dx}{dt} \Rightarrow z\,\frac{dz}{dt} = x\,\frac{dx}{dt} + 2\pi\sin\!\left(\frac{\pi}{2}x\right)\cos\!\left(\frac{\pi}{2}x\right)\frac{dx}{dt}.$$ When

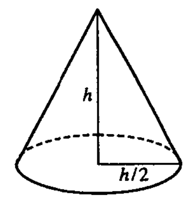
$$(x,y) = \left(\frac{1}{3},1\right),\ z = \sqrt{\left(\frac{1}{3}\right)^2 + 1^2} = \sqrt{\frac{10}{9}} = \frac{1}{3}\sqrt{10},\text{ so } \frac{1}{3}\sqrt{10}\,\frac{dz}{dt} = \frac{1}{3}\sqrt{10} + 2\pi\sin\frac{\pi}{6}\cos\frac{\pi}{6}\cdot\sqrt{10} \Rightarrow$$

$$\frac{1}{3}\frac{dz}{dt} = \frac{1}{3} + 2\pi\left(\frac{1}{2}\right)\left(\frac{1}{2}\sqrt{3}\right) \Rightarrow \frac{dz}{dt} = 1 + \frac{3\sqrt{3}\,\pi}{2}\text{ cm/s.}$$

27. We are given that $\frac{dV}{dt} = 30\text{ ft}^3/\text{min}$. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12} \Rightarrow$

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 30 = \frac{\pi h^2}{4}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{120}{\pi h^2}.$$

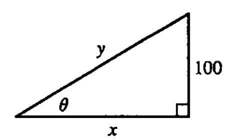
When $h = 10$ ft, $\frac{dh}{dt} = \frac{120}{10^2\pi} = \frac{6}{5\pi} \approx 0.38$ ft/min.



28. We are given $dx/dt = 8$ ft/s. $\cot\theta = \frac{x}{100} \Rightarrow x = 100\cot\theta \Rightarrow$

$$\frac{dx}{dt} = -100\csc^2\theta\,\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{\sin^2\theta}{100}\cdot 8.$$ When $y = 200$, $\sin\theta = \frac{100}{200} = \frac{1}{2} \Rightarrow$

$$\frac{d\theta}{dt} = -\frac{(1/2)^2}{100}\cdot 8 = -\frac{1}{50}\text{ rad/s.}$$ The angle is decreasing at a rate of $\frac{1}{50}$ rad/s.



35. With $R_1 = 80$ and $R_2 = 100$, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80} + \frac{1}{100} = \frac{180}{8000} = \frac{9}{400}$, so $R = \frac{400}{9}$. Differentiating $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ with respect to t, we have $-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{R_1^2}\frac{dR_1}{dt} - \frac{1}{R_2^2}\frac{dR_2}{dt} \Rightarrow \frac{dR}{dt} = R^2\left(\frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}\right)$. When $R_1 = 80$ and $R_2 = 100$,

$$\frac{dR}{dt} = \frac{400^2}{9^2}\left[\frac{1}{80^2}(0.3) + \frac{1}{100^2}(0.2)\right] = \frac{107}{810} \approx 0.132\ \Omega/\text{s}.$$