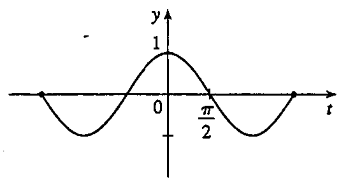
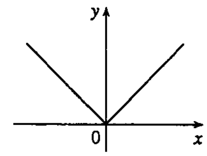
# Homework 17

## Section 4.1:

**22.** $f(t) = \cos t$, $-\frac{3\pi}{2} \le t \le \frac{3\pi}{2}$. Absolute and local maximum $f(0) = 1$; absolute and local minima $f(\pm\pi, -1)$.




**24.** $f(x) = | x |$. No absolute or local maximum. Absolute and local minimum $f(0) = 0$.




**30.** $f(x) = x^3 + 6x^2 - 15x \;\Rightarrow\; f'(x) = 3x^2 + 12x - 15 = 3(x^2 + 4x - 5) = 3(x+5)(x-1)$.
$f'(x) = 0 \;\Rightarrow\; x = -5, 1$. These are the only critical numbers.

**42.** $h(t) = 3t - \arcsin t \;\Rightarrow\; h'(t) = 3 - \dfrac{1}{\sqrt{1-t^2}}$. $h'(t) = 0 \;\Rightarrow\; 3 = \dfrac{1}{\sqrt{1-t^2}} \;\Rightarrow\; \sqrt{1-t^2} = \frac{1}{3} \;\Rightarrow$
$1 - t^2 = \frac{1}{9} \;\Rightarrow\; t^2 = \frac{8}{9} \;\Rightarrow\; t = \pm\frac{2}{3}\sqrt{2} \approx \pm 0.94$, both in the domain of $h$, which is $[-1, 1]$.

**48.** $f(x) = 5 + 54x - 2x^3$, $[0, 4]$. $f'(x) = 54 - 6x^2 = 6(9 - x^2) = 6(3+x)(3-x) = 0 \;\Leftrightarrow\; x = -3, 3$. $f(0) = 5$, $f(3) = 113$, and $f(4) = 93$. So $f(3) = 113$ is the absolute maximum value and $f(0) = 5$ is the absolute minimum value.

**54.** $f(x) = \dfrac{x}{x^2 - x + 1}$, $[0, 3]$.

$$f'(x) = \frac{(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2} = \frac{x^2 - x + 1 - 2x^2 + x}{(x^2 - x + 1)^2} = \frac{1 - x^2}{(x^2 - x + 1)^2} = \frac{(1+x)(1-x)}{(x^2 - x + 1)^2} = 0 \;\Leftrightarrow\; x = \pm 1,$$

but $x = -1$ is not in the given interval, $[0, 3]$. $f(0) = 0$, $f(1) = 1$, and $f(3) = \frac{3}{7}$. So $f(1) = 1$ is the absolute maximum value and $f(0) = 0$ is the absolute minimum value.

**58.** $f(t) = t + \cot(t/2)$, $[\pi/4, 7\pi/4]$. $f'(t) = 1 - \csc^2(t/2) \cdot \frac{1}{2}$.
$f'(t) = 0 \;\Rightarrow\; \frac{1}{2}\csc^2(t/2) = 1 \;\Rightarrow\; \csc^2(t/2) = 2 \;\Rightarrow\; \csc(t/2) = \pm\sqrt{2} \;\Rightarrow\; \frac{1}{2}t = \frac{\pi}{4}$ or $\frac{1}{2}t = \frac{3\pi}{4}$
$\left[\frac{\pi}{4} \le t \le \frac{7\pi}{4} \;\Rightarrow\; \frac{\pi}{8} \le \frac{1}{2}t \le \frac{7\pi}{8}\right.$ and $\csc(t/2) \ne -\sqrt{2}$ in the last interval$\left.\right] \;\Rightarrow\; t = \frac{\pi}{2}$ or $t = \frac{3\pi}{2}$.
$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + \cot\frac{\pi}{8} \approx 3.20$, $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \cot\frac{\pi}{4} = \frac{\pi}{2} + 1 \approx 2.57$, $f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} + \cot\frac{3\pi}{4} = \frac{3\pi}{2} - 1 \approx 3.71$, and $f\left(\frac{7\pi}{4}\right) = \frac{7\pi}{4} + \cot\frac{7\pi}{8} \approx 3.08$. So $f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - 1$ is the absolute maximum value and $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 1$ is the absolute minimum value.

**60.** $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$. $f'(x) = 0 \;\Rightarrow\; x = 1$. [Note that 0 is not in the domain of $f$.]
$f\left(\frac{1}{2}\right) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.