Homework 19

Section 4.3:

8. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2,4)$ and $(6,9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is concave upward. This happens on $(1,3)$, $(5,7)$, and $(8,9)$. Similarly, f is concave downward when f' is decreasing—that is, on $(0,1)$, $(3,5)$, and $(7,8)$.

(d) f has inflection points at $x = 1, 3, 5, 7,$ and 8, since the direction of concavity changes at each of these values.

20. $f(x) = \dfrac{x^2}{x-1} \quad \Rightarrow \quad f'(x) = \dfrac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}.$

First Derivative Test: $f'(x) > 0 \Rightarrow x < 0$ or $x > 2$ and $f'(x) < 0 \Rightarrow 0 < x < 1$ or $1 < x < 2$. Since f' changes from positive to negative at $x = 0$, $f(0) = 0$ is a local maximum value; and since f' changes from negative to positive at $x = 2$, $f(2) = 4$ is a local minimum value.

Second Derivative Test:

$$f''(x) = \frac{(x-1)^2(2x-2) - (x^2-2x)2(x-1)}{[(x-1)^2]^2} = \frac{2(x-1)[(x-1)^2 - (x^2-2x)]}{(x-1)^4} = \frac{2}{(x-1)^3}.$$

$f'(x) = 0 \Leftrightarrow x = 0, 2$. $f''(0) = -2 < 0 \Rightarrow f(0) = 0$ is a local maximum value. $f''(2) = 2 > 0 \Rightarrow f(2) = 4$ is a local minimum value.

Preference: Since calculating the second derivative is fairly difficult, the First Derivative Test is easier to use for this function.

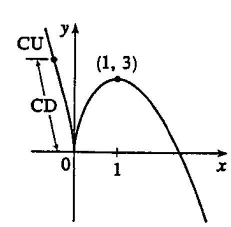
40. (a) $G(x) = 5x^{2/3} - 2x^{5/3} \quad \Rightarrow \quad G'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3} = \frac{10}{3}x^{-1/3}(1-x) = \dfrac{10(1-x)}{3x^{1/3}}.$

$G'(x) > 0 \Leftrightarrow 0 < x < 1$ and $G'(x) < 0 \Leftrightarrow x < 0$ or $x > 1$. So G is increasing on $(0,1)$ and G is decreasing on $(-\infty, 0)$ and $(1, \infty)$.

(b) G changes from decreasing to increasing at $x = 0$, so $G(0) = 0$ is a local minimum value. G changes from increasing to decreasing at $x = 1$, so $G(1) = 3$ is a local maximum value. Note that the First Derivative Test applies at $x = 0$ even though G' is not defined at $x = 0$, since G is continuous at 0.

(c) $G''(x) = -\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3} = -\frac{10}{9}x^{-4/3}(1+2x)$. $G''(x) > 0 \Leftrightarrow x < -\frac{1}{2}$ and $G''(x) < 0 \Leftrightarrow -\frac{1}{2} < x < 0$ or $x > 0$. So G is CU on $(-\infty, -\frac{1}{2})$ and G is CD on $(-\frac{1}{2}, 0)$ and $(0, \infty)$. The only change in concavity occurs at $x = -\frac{1}{2}$, so there is an inflection point at $(-\frac{1}{2}, 6/\sqrt[3]{4})$.

(d)



42. (a) $f(x) = \ln(x^4 + 27) \;\Rightarrow\; f'(x) = \dfrac{4x^3}{x^4 + 27}$. $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$, so f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) $f(0) = \ln 27 \approx 3.3$ is a local minimum value.

(c) $$f''(x) = \frac{(x^4 + 27)(12x^2) - 4x^3(4x^3)}{(x^4 + 27)^2} = \frac{4x^2\left[3(x^4 + 27) - 4x^4\right]}{(x^4 + 27)^2} = \frac{4x^2(81 - x^4)}{(x^4 + 27)^2} = \frac{-4x^2(x^2 + 9)(x + 3)(x - 3)}{(x^4 + 27)^2}$$

$f''(x) > 0$ if $-3 < x < 0$ and $0 < x < 3$, and $f''(x) < 0$ if $x < -3$ or $x > 3$. Thus, f is concave upward on $(-3, 0)$ and $(0, 3)$ [hence on $(-3, 3)$] and f is concave downward on $(-\infty, -3)$ and $(3, \infty)$. There are inflection points at $(\pm 3, \ln 108) \approx (\pm 3, 4.68)$.

(d)