Homework 20

Section 4.3:

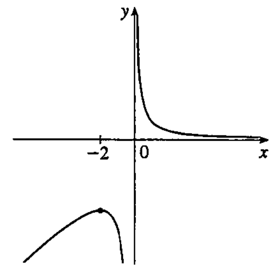
24. Vertical asymptote $x = 0$

$f'(x) > 0$ if $x < -2 \;\Rightarrow\;$ f is increasing on $(-\infty, -2)$.

$f'(x) < 0$ if $x > -2$ $(x \neq 0)$ $\;\Rightarrow\;$ f is decreasing on $(-2, 0)$ and $(0, \infty)$.

$f''(x) < 0$ if $x < 0 \;\Rightarrow\;$ f is concave downward on $(-\infty, 0)$.

$f''(x) > 0$ if $x > 0 \;\Rightarrow\;$ f is concave upward on $(0, \infty)$.

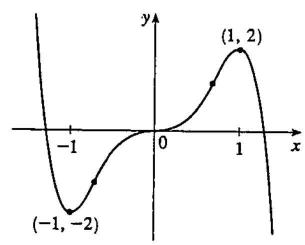


38. (a) $h(x) = 5x^3 - 3x^5 \;\Rightarrow\; h'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2) = 15x^2(1 + x)(1 - x)$. $h'(x) > 0 \;\Leftrightarrow\;$ $-1 < x < 0$ and $0 < x < 1$ [note that $h'(0) = 0$] and $h'(x) < 0 \;\Leftrightarrow\; x < -1$ or $x > 1$. So h is increasing on $(-1, 1)$ and h is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

(b) h changes from decreasing to increasing at $x = -1$, so $h(-1) = -2$ is a local minimum value. h changes from increasing to decreasing at $x = 1$, so $h(1) = 2$ is a local maximum value.

(c) $h''(x) = 30x - 60x^3 = 30x(1 - 2x^2)$. $h''(x) = 0 \;\Leftrightarrow\; x = 0$ or $1 - 2x^2 = 0 \;\Leftrightarrow\; x = 0$ or $x = \pm 1/\sqrt{2}$. $h''(x) > 0$ on $\left(-\infty, -1/\sqrt{2}\right)$ and $\left(0, 1/\sqrt{2}\right)$, and $h''(x) < 0$ on $\left(-1/\sqrt{2}, 0\right)$ and $\left(1/\sqrt{2}, \infty\right)$. So h is CU on $\left(-\infty, -1/\sqrt{2}\right)$ and $\left(0, 1/\sqrt{2}\right)$, and h is CD on $\left(-1/\sqrt{2}, 0\right)$ and $\left(1/\sqrt{2}, \infty\right)$. There are inflection points at $\left(-1/\sqrt{2}, -7/(4\sqrt{2})\right)$, $(0, 0)$, and $\left(1/\sqrt{2}, 7/(4\sqrt{2})\right)$.

(d)

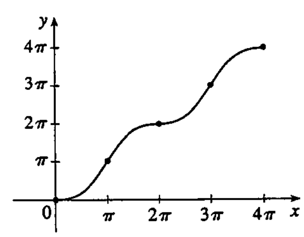


44. (a) $S(x) = x - \sin x$, $0 \le x \le 4\pi \;\Rightarrow\; S'(x) = 1 - \cos x$. $S'(x) = 0 \;\Leftrightarrow\; \cos x = 1 \;\Leftrightarrow\; x = 0, 2\pi$, and 4π. $S'(x) > 0 \;\Leftrightarrow\; \cos x < 1$, which is true for all x except integer multiples of 2π, so S is increasing on $(0, 4\pi)$ since $S'(2\pi) = 0$.

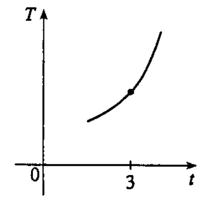
(b) There is no local maximum or minimum.

(c) $S''(x) = \sin x$. $S''(x) > 0$ if $0 < x < \pi$ or $2\pi < x < 3\pi$, and $S''(x) < 0$ if $\pi < x < 2\pi$ or $3\pi < x < 4\pi$. So S is CU on $(0, \pi)$ and $(2\pi, 3\pi)$, and S is CD on $(\pi, 2\pi)$ and $(3\pi, 4\pi)$. There are inflection points at (π, π), $(2\pi, 2\pi)$, and $(3\pi, 3\pi)$.

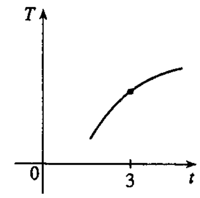
(d)



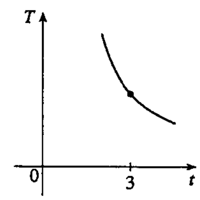
62. (a) I'm very unhappy. It's uncomfortably hot and $f'(3) = 2$ indicates that the temperature is increasing, and $f''(3) = 4$ indicates that the rate of increase is increasing. (The temperature is rapidly getting warmer.)



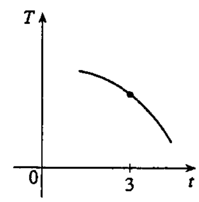
(b) I'm still unhappy, but not as unhappy as in part (a). It's uncomfortably hot and $f'(3) = 2$ indicates that the temperature is increasing, but $f''(3) = -4$ indicates that the rate of increase is decreasing. (The temperature is slowly getting warmer.)



(c) I'm somewhat happy. It's uncomfortably hot and $f'(3) = -2$ indicates that the temperature is decreasing, but $f''(3) = 4$ indicates that the rate of change is increasing. (The rate of change is negative but it's becoming less negative. The temperature is slowly getting cooler.)



(d) I'm very happy. It's uncomfortably hot and $f'(3) = -2$ indicates that the temperature is decreasing, and $f''(3) = -4$ indicates that the rate of change is decreasing, that is, becoming more negative. (The temperature is rapidly getting cooler.)



82. $f(x) = x^4 \;\Rightarrow\; f'(x) = 4x^3 \;\Rightarrow\; f''(x) = 12x^2 \;\Rightarrow\; f''(0) = 0$. For $x < 0$, $f''(x) > 0$, so f is CU on $(-\infty, 0)$; for $x > 0$, $f''(x) > 0$, so f is also CU on $(0, \infty)$. Since f does not change concavity at 0, $(0, 0)$ is not an inflection point.