Homework 21

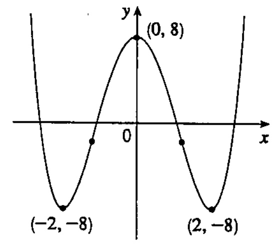
Section 4.5:

4. $y = f(x) = x^4 - 8x^2 + 8$ **A.** $D = \mathbb{R}$ **B.** y-intercept $f(0) = 8$; x-intercepts: $f(x) = 0 \Rightarrow$ [by the quadratic formula] $x = \pm\sqrt{4 \pm 2\sqrt{2}} \approx \pm 2.61, \pm 1.08$ **C.** $f(-x) = f(x)$, so f is even and symmetric about the y-axis **D.** No asymptote **E.** $f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x+2)(x-2) > 0 \Leftrightarrow -2 < x < 0$ or $x > 2$, so f is increasing on $(-2, 0)$ and $(2, \infty)$, and f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

F. Local maximum value $f(0) = 8$, local minimum values $f(\pm 2) = -8$

G. $f''(x) = 12x^2 - 16 = 4(3x^2 - 4) > 0 \Rightarrow |x| > 2/\sqrt{3}$ [≈ 1.15], so f is CU on $\left(-\infty, -2/\sqrt{3}\right)$ and $\left(2/\sqrt{3}, \infty\right)$, and f is CD on $\left(-2/\sqrt{3}, 2/\sqrt{3}\right)$. IP at $\left(\pm 2/\sqrt{3}, -\frac{8}{9}\right)$

H.



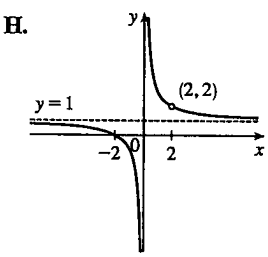
10. $y = f(x) = \dfrac{x^2 - 4}{x^2 - 2x} = \dfrac{(x+2)(x-2)}{x(x-2)} = \dfrac{x+2}{x} = 1 + \dfrac{2}{x}$ for $x \neq 2$. There is a hole in the graph at $(2, 2)$.

A. $D = \{x \mid x \neq 0, 2\} = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$ **B.** y-intercept: none; x-intercept: -2 **C.** No symmetry

D. $\lim\limits_{x \to \pm\infty} \dfrac{x+2}{x} = 1$, so $y = 1$ is a HA. $\lim\limits_{x \to 0^-} \dfrac{x+2}{x} = -\infty$, $\lim\limits_{x \to 0^+} \dfrac{x+2}{x} = \infty$, so $x = 0$ is a VA. **E.** $f'(x) = -2/x^2 < 0$ [$x \neq 0, 2$] so f is decreasing on $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. **F.** No extrema

G. $f''(x) = 4/x^3 > 0 \Leftrightarrow x > 0$, so f is CU on $(0, 2)$ and $(2, \infty)$ and CD on $(-\infty, 0)$. No IP

H.



28. $y = f(x) = x/\sqrt{x^2 - 1}$ **A.** $D = (-\infty, -1) \cup (1, \infty)$ **B.** No intercepts **C.** $f(-x) = -f(x)$, so f is odd; the graph is symmetric about the origin. **D.** $\lim\limits_{x \to \infty} \dfrac{x}{\sqrt{x^2 - 1}} = 1$ and $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2 - 1}} = -1$, so $y = \pm 1$ are HA. $\lim\limits_{x \to 1^+} f(x) = +\infty$ and $\lim\limits_{x \to -1^-} f(x) = -\infty$, so $x = \pm 1$ are VA.

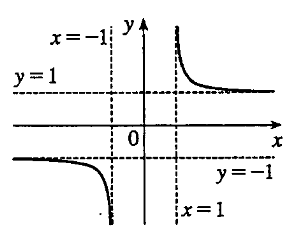
E. $f'(x) = \dfrac{\sqrt{x^2 - 1} - x \cdot \dfrac{x}{\sqrt{x^2 - 1}}}{[(x^2 - 1)^{1/2}]^2} = \dfrac{x^2 - 1 - x^2}{(x^2 - 1)^{3/2}} = \dfrac{-1}{(x^2 - 1)^{3/2}} < 0$, so f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

F. No extreme values

G. $f''(x) = (-1)\left(-\frac{3}{2}\right)(x^2 - 1)^{-5/2} \cdot 2x = \dfrac{3x}{(x^2 - 1)^{5/2}}$.

$f''(x) < 0$ on $(-\infty, -1)$ and $f''(x) > 0$ on $(1, \infty)$, so f is CD on $(-\infty, -1)$ and CU on $(1, \infty)$. No IP

H.



Section 4.4:

12. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\tan 5x} \overset{\text{H}}{=} \lim\limits_{x\to 0}\dfrac{4\cos 4x}{5\sec^2(5x)} = \dfrac{4(1)}{5(1)^2} = \dfrac{4}{5}$

22. This limit has the form $\frac{0}{0}$. $\lim\limits_{t\to 0}\dfrac{8^t - 5^t}{t} \overset{\text{H}}{=} \lim\limits_{t\to 0}\dfrac{8^t\ln 8 - 5^t\ln 5}{1} = \ln 8 - \ln 5 = \ln\dfrac{8}{5}$

36. $\lim\limits_{x\to 0^+}\dfrac{x^x - 1}{\ln x + x - 1}$. From Example 9, $\lim\limits_{x\to 0^+} x^x = 1$, so $\lim\limits_{x\to 0^+}(x^x - 1) = 0$. As $x\to 0^+$, $\ln x \to -\infty$, so $\ln x + x - 1 \to -\infty$ as $x \to 0^+$. Thus, $\lim\limits_{x\to 0^+}\dfrac{x^x - 1}{\ln x + x - 1} = 0$.

44. This limit has the form $0\cdot(-\infty)$.

$$\lim_{x\to 0^+}\sin x\,\ln x = \lim_{x\to 0^+}\frac{\ln x}{\csc x} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{1/x}{-\csc x\,\cot x} = -\lim_{x\to 0^+}\left(\frac{\sin x}{x}\cdot\tan x\right) = -\left(\lim_{x\to 0^+}\frac{\sin x}{x}\right)\left(\lim_{x\to 0^+}\tan x\right)$$
$$= -1\cdot 0 = 0$$

52. This limit has the form $\infty - \infty$.

$$\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{\cos x}{\sin x} - \frac{1}{x}\right) = \lim_{x\to 0}\frac{x\cos x - \sin x}{x\sin x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{x(-\sin x) + \cos x - \cos x}{x\cos x + \sin x}$$
$$= -\lim_{x\to 0}\frac{x\sin x}{x\cos x + \sin x} \overset{\text{H}}{=} -\lim_{x\to 0}\frac{x\cos x + \sin x}{x(-\sin x) + \cos x + \cos x} = -\frac{0+0}{0+1+1} = 0$$