Section 4.4:

42. This limit has the form $\infty \cdot 0$. We'll change it to the form $\frac{\infty}{\infty}$.

$$\lim_{x \to \infty} \sqrt{x}\, e^{-x/2} = \lim_{x \to \infty} \frac{\sqrt{x}}{e^{x/2}} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{1}{2}e^{x/2}} = \lim_{x \to \infty} \frac{1}{\sqrt{x}\, e^{x/2}} = 0$$

46. This limit has the form $\infty \cdot 0$.

$$\lim_{x \to \infty} x \tan(1/x) = \lim_{x \to \infty} \frac{\tan(1/x)}{1/x} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{\sec^2(1/x)(-1/x^2)}{-1/x^2} = \lim_{x \to \infty} \sec^2(1/x) = 1^2 = 1$$

56. $y = (\tan 2x)^x \;\Rightarrow\; \ln y = x \cdot \ln \tan 2x$, so

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} x \cdot \ln \tan 2x = \lim_{x \to 0^+} \frac{\ln \tan 2x}{1/x} \overset{\text{H}}{=} \lim_{x \to 0^+} \frac{(1/\tan 2x)(2\sec^2 2x)}{-1/x^2} = \lim_{x \to 0^+} \frac{-2x^2 \cos 2x}{\sin 2x \cos^2 2x}$$

$$= \lim_{x \to 0^+} \frac{2x}{\sin 2x} \cdot \lim_{x \to 0^+} \frac{-x}{\cos 2x} = 1 \cdot 0 = 0 \;\Rightarrow\;$$

$$\lim_{x \to 0^+} (\tan 2x)^x = \lim_{x \to 0^+} e^{\ln y} = e^0 = 1.$$

58. $y = \left(1 + \dfrac{a}{x}\right)^{bx} \;\Rightarrow\; \ln y = bx \ln\left(1 + \dfrac{a}{x}\right)$, so

$$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{b\ln(1 + a/x)}{1/x} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{b\left(\dfrac{1}{1 + a/x}\right)\left(-\dfrac{a}{x^2}\right)}{-1/x^2} = \lim_{x \to \infty} \frac{ab}{1 + a/x} = ab \;\Rightarrow\;$$

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx} = \lim_{x \to \infty} e^{\ln y} = e^{ab}.$$

62. $y = (e^x + x)^{1/x} \;\Rightarrow\; \ln y = \dfrac{1}{x}\ln(e^x + x)$,

so $\displaystyle\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln(e^x + x)}{x} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{e^x + 1}{e^x + x} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{e^x}{e^x + 1} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{e^x}{e^x} = 1 \;\Rightarrow\;$

$$\lim_{x \to \infty} (e^x + x)^{1/x} = \lim_{x \to \infty} e^{\ln y} = e^1 = e.$$

72. This limit has the form $\frac{\infty}{\infty}$. $\displaystyle\lim_{x \to \infty} \frac{\ln x}{x^p} \overset{\text{H}}{=} \lim_{x \to \infty} \frac{1/x}{px^{p-1}} = \lim_{x \to \infty} \frac{1}{px^p} = 0$ since $p > 0$.

74. $\displaystyle\lim_{x \to (\pi/2)^-} \frac{\sec x}{\tan x} \overset{\text{H}}{=} \lim_{x \to (\pi/2)^-} \frac{\sec x \tan x}{\sec^2 x} = \lim_{x \to (\pi/2)^-} \frac{\tan x}{\sec x}.$ Repeated applications of l'Hospital's Rule result in the

original limit or the limit of the reciprocal of the function. Another method is to simplify first:

$$\lim_{x \to (\pi/2)^-} \frac{\sec x}{\tan x} = \lim_{x \to (\pi/2)^-} \frac{1/\cos x}{\sin x/\cos x} = \lim_{x \to (\pi/2)^-} \frac{1}{\sin x} = \frac{1}{1} = 1$$