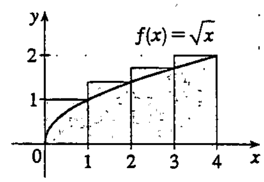
Homework 24

Section 5.1:

4. (a) $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \qquad \left[\Delta x = \frac{4-0}{4} = 1\right]$

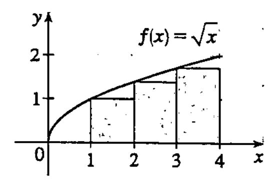
$$= f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1 + f(x_4)\cdot 1$$
$$= f(1) + f(2) + f(3) + f(4)$$
$$= \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} \approx 6.1463$$



Since f is *increasing* on $[0, 4]$, R_4 is an *overestimate.*

(b) $L_4 = \sum_{i=1}^{4} f(x_{i-1})\,\Delta x = f(x_0)\cdot 1 + f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1$

$$= f(0) + f(1) + f(2) + f(3)$$
$$= \sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} \approx 4.1463$$



Since f is *increasing* on $[0, 4]$, L_4 is an *underestimate.*

14. (a) $d \approx L_5 = (30 \text{ ft/s})(12 \text{ s}) + 28 \cdot 12 + 25 \cdot 12 + 22 \cdot 12 + 24 \cdot 12$
$$= (30 + 28 + 25 + 22 + 24) \cdot 12 = 129 \cdot 12 = 1548 \text{ ft}$$

(b) $d \approx R_5 = (28 + 25 + 22 + 24 + 27) \cdot 12 = 126 \cdot 12 = 1512 \text{ ft}$

(c) The estimates are neither lower nor upper estimates since v is neither an increasing nor a decreasing function of t.

18. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate. We will use M_6 to get an estimate. $\Delta t = \frac{30-0}{6} = 5 \text{ s} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$M_6 = \tfrac{1}{720}[v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)]$$
$$= \tfrac{1}{720}(31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \tfrac{1}{720}(521.75) \approx 0.725 \text{ km}$$

For a very rough check on the above calculation, we can draw a line from $(0, 0)$ to $(30, 120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

Section 5.2:

6. (a) $\int_{-2}^{4} g(x)\,dx \approx R_6 = [g(-1)+g(0)+g(1)+g(2)+g(3)+g(4)]\,\Delta x$

$= [-\frac{3}{2}+0+\frac{3}{2}+\frac{1}{2}+(-1)+\frac{1}{2}](1) = 0$

(b) $\int_{-2}^{4} g(x)\,dx \approx L_6 = [g(-2)+g(-1)+g(0)+g(1)+g(2)+g(3)]\,\Delta x$

$= [0+(-\frac{3}{2})+0+\frac{3}{2}+\frac{1}{2}+(-1)](1) = -\frac{1}{2}$

(c) $\int_{-2}^{4} g(x)\,dx \approx M_6 = [g(-\frac{3}{2})+g(-\frac{1}{2})+g(\frac{1}{2})+g(\frac{3}{2})+g(\frac{5}{2})+g(\frac{7}{2})]\,\Delta x$

$= [-1+(-1)+1+1+0+(-\frac{1}{2})](1) = -\frac{1}{2}$

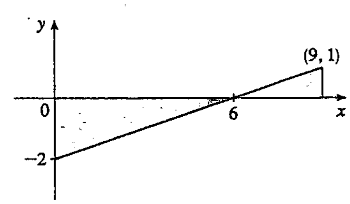
34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

36. $\int_0^9 \left(\frac{1}{3}x - 2\right) dx$ can be interpreted as the difference of the areas of the two shaded triangles; that is, $-\frac{1}{2}(6)(2) + \frac{1}{2}(3)(1) = -6 + \frac{3}{2} = -\frac{9}{2}$.



40. $\int_0^{10} |x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2(\frac{1}{2})(5)(5) = 25$.