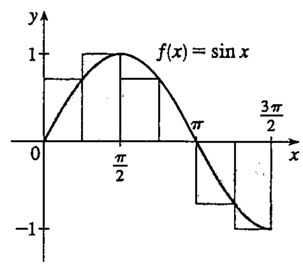
Homework 25

Section 5.2:

4. (a) $f(x) = \sin x,\ 0 \le x \le \frac{3\pi}{2}$. $\Delta x = \frac{b-a}{n} = \frac{\frac{3\pi}{2} - 0}{6} = \frac{\pi}{4}$.

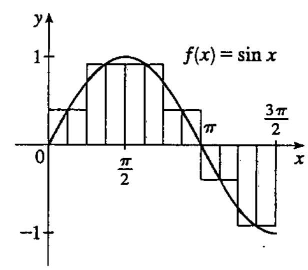
Since we are using right endpoints, $x_i^* = x_i$.



$$\begin{aligned}
R_6 &= \sum_{i=1}^{6} f(x_i)\,\Delta x \\
&= (\Delta x)[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)] \\
&= \frac{\pi}{4}\left[f\left(\frac{\pi}{4}\right) + f\left(\frac{2\pi}{4}\right) + f\left(\frac{3\pi}{4}\right) + f\left(\frac{4\pi}{4}\right) + f\left(\frac{5\pi}{4}\right) + f\left(\frac{6\pi}{4}\right)\right] \\
&= \frac{\pi}{4}\left(\sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi + \sin\frac{5\pi}{4} + \sin\frac{3\pi}{2}\right) \\
&= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0 - \frac{\sqrt{2}}{2} - 1\right) = \frac{\pi\sqrt{2}}{8} \approx 0.555360
\end{aligned}$$

The Riemann sum represents the sum of the areas of the three rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis.

(b) Since we are using midpoints, $x_i^* = \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$.



$$\begin{aligned}
M_6 &= \sum_{i=1}^{6} f(\overline{x}_i)\,\Delta x \\
&= (\Delta x)\,[f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + f(\overline{x}_4) + f(\overline{x}_5) + f(\overline{x}_6)] \\
&= \frac{\pi}{4}\left[f\left(\frac{\pi}{8}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{7\pi}{8}\right) + f\left(\frac{9\pi}{8}\right) + f\left(\frac{11\pi}{8}\right)\right] \\
&= \frac{\pi}{4}\left(\sin\frac{\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} + \sin\frac{7\pi}{8} + \sin\frac{9\pi}{8} + \sin\frac{11\pi}{8}\right) \\
&\approx \tfrac{\pi}{4}(1.306563) \approx 1.026172
\end{aligned}$$

The Riemann sum represents the sum of the areas of the four rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis. Note that the Riemann sum has the same value as the sum of the areas of the first two rectangles.

8. (a) Using the right endpoints to approximate $\int_3^9 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_i)\,\Delta x = 2[f(5) + f(7) + f(9)] = 2\,(-0.6 + 0.9 + 1.8) = 4.2.$$

Since f is *increasing*, using *right* endpoints gives an *overestimate.*

(b) Using the left endpoints to approximate $\int_3^9 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_{i-1})\,\Delta x = 2[f(3) + f(5) + f(7)] = 2\,(-3.4 - 0.6 + 0.9) = -6.2.$$

Since f is *increasing*, using *left* endpoints gives an *underestimate.*

(c) Using the midpoint of each interval to approximate $\int_3^9 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(\overline{x}_i)\,\Delta x = 2[f(4) + f(6) + f(8)] = 2\,(-2.1 + 0.3 + 1.4) = -0.8.$$

We cannot say anything about the midpoint estimate compared to the exact value of the integral.

9. $\Delta x = (8-0)/4 = 2$, so the endpoints are 0, 2, 4, 6, and 8, and the midpoints are 1, 3, 5, and 7. The Midpoint Rule gives

$$\int_0^8 \sin\sqrt{x}\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 2\left(\sin\sqrt{1} + \sin\sqrt{3} + \sin\sqrt{5} + \sin\sqrt{7}\right) \approx 2(3.0910) = 6.1820.$$

10. $\Delta x = (\pi/2 - 0)/4 = \frac{\pi}{8}$, so the endpoints are $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$, and $\frac{\pi}{2}$, and the midpoints are $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}$, and $\frac{7\pi}{16}$. The Midpoint Rule gives

$$\int_0^{\pi/2} \cos^4 x\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = \tfrac{\pi}{8}\left[\cos^4\left(\tfrac{\pi}{16}\right) + \cos^4\left(\tfrac{3\pi}{16}\right) + \cos^4\left(\tfrac{5\pi}{16}\right) + \cos^4\left(\tfrac{7\pi}{16}\right)\right] = \tfrac{\pi}{8}\left(\tfrac{3}{2}\right) \approx 0.5890.$$

12. $\Delta x = (5-1)/4 = 1$, so the endpoints are 1, 2, 3, 4, and 5, and the midpoints are 1.5, 2.5, 3.5, and 4.5. The Midpoint Rule gives

$$\int_1^5 x^2 e^{-x}\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 1\left[(1.5)^2 e^{-1.5} + (2.5)^2 e^{-2.5} + (3.5)^2 e^{-3.5} + (4.5)^2 e^{-4.5}\right] \approx 1.6099.$$

44. $\int_1^3 (2e^x - 1)\,dx = 2\int_1^3 e^x\,dx - \int_1^3 1\,dx = 2(e^3 - e) - 1(3-1) = 2e^3 - 2e - 2$

52. $F(0) = \int_2^0 f(t)\,dt = -\int_0^2 f(t)\,dt$, so $F(0)$ is negative, and similarly, so is $F(1)$. $F(3)$ and $F(4)$ are negative since they represent negatives of areas below the x-axis. Since $F(2) = \int_2^2 f(t)\,dt = 0$ is the only non-negative value, choice C is the largest.

Appendix E:

5. $\sum_{k=0}^{4} \frac{2k-1}{2k+1} = -1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$

6. $\sum_{k=5}^{8} x^k = x^5 + x^6 + x^7 + x^8$

7. $\sum_{i=1}^{n} i^{10} = 1^{10} + 2^{10} + 3^{10} + \cdots + n^{10}$

8. $\sum_{j=n}^{n+3} j^2 = n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2$

9. $\sum_{j=0}^{n-1} (-1)^j = 1 - 1 + 1 - 1 + \cdots + (-1)^{n-1}$

10. $\sum_{i=1}^{n} f(x_i)\,\Delta x_i = f(x_1)\,\Delta x_1 + f(x_2)\,\Delta x_2 + f(x_3)\,\Delta x_3 + \cdots + f(x_n)\,\Delta x_n$

11. $1 + 2 + 3 + 4 + \cdots + 10 = \sum_{i=1}^{10} i$

12. $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} = \sum_{i=3}^{7} \sqrt{i}$

13. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{19}{20} = \sum_{i=1}^{19} \frac{i}{i+1}$

14. $\frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \frac{6}{10} + \cdots + \frac{23}{27} = \sum_{i=3}^{23} \frac{i}{i+4}$

15. $2 + 4 + 6 + 8 + \cdots + 2n = \sum_{i=1}^{n} 2i$

16. $1 + 3 + 5 + 7 + \cdots + (2n-1) = \sum_{i=1}^{n} (2i-1)$

17. $1 + 2 + 4 + 8 + 16 + 32 = \sum_{i=0}^{5} 2^i$

18. $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} = \sum_{i=1}^{6} \frac{1}{i^2}$

19. $x + x^2 + x^3 + \cdots + x^n = \sum_{i=1}^{n} x^i$

20. $1 - x + x^2 - x^3 + \cdots + (-1)^n x^n = \sum_{i=0}^{n} (-1)^i x^i$