

Homework 4

Section 2.3:

$$\begin{aligned} 6. \lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6} &= \sqrt{\lim_{u \rightarrow -2} (u^4 + 3u + 6)} && [11] \\ &= \sqrt{\lim_{u \rightarrow -2} u^4 + 3 \lim_{u \rightarrow -2} u + \lim_{u \rightarrow -2} 6} && [1, 2, \text{ and } 3] \\ &= \sqrt{(-2)^4 + 3(-2) + 6} && [9, 8, \text{ and } 7] \\ &= \sqrt{16 - 6 + 6} = \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} 8. \lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right)^2 &= \left(\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^3 - 3t + 5} \right)^2 && [\text{Limit Law 6}] \\ &= \left(\frac{\lim_{t \rightarrow 2} (t^2 - 2)}{\lim_{t \rightarrow 2} (t^3 - 3t + 5)} \right)^2 && [5] \\ &= \left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - 3 \lim_{t \rightarrow 2} t + \lim_{t \rightarrow 2} 5} \right)^2 && [1, 2, \text{ and } 3] \\ &= \left(\frac{4 - 2}{8 - 3(2) + 5} \right)^2 && [9, 7, \text{ and } 8] \\ &= \left(\frac{2}{7} \right)^2 = \frac{4}{49} \end{aligned}$$

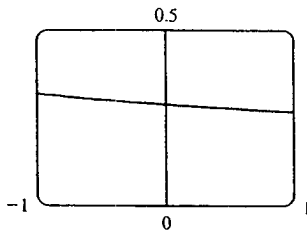
10. (a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \rightarrow 2$, just as in Example 3. Remember that in finding $\lim_{x \rightarrow a} f(x)$, we never consider $x = a$.

$$\begin{aligned} 22. \lim_{u \rightarrow 2} \frac{\sqrt{4u+1} - 3}{u-2} &= \lim_{u \rightarrow 2} \frac{\sqrt{4u+1} - 3}{u-2} \cdot \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3} = \lim_{u \rightarrow 2} \frac{(\sqrt{4u+1})^2 - 3^2}{(u-2)(\sqrt{4u+1} + 3)} \\ &= \lim_{u \rightarrow 2} \frac{4u + 1 - 9}{(u-2)(\sqrt{4u+1} + 3)} = \lim_{u \rightarrow 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1} + 3)} \\ &= \lim_{u \rightarrow 2} \frac{4}{\sqrt{4u+1} + 3} = \frac{4}{\sqrt{9} + 3} = \frac{2}{3} \end{aligned}$$

$$26. \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t(t+1)} \right) = \lim_{t \rightarrow 0} \frac{t+1-1}{t(t+1)} = \lim_{t \rightarrow 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$$

34. (a)



$$\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \approx 0.29$$

(b)

x	$f(x)$
-0.001	0.2886992
-0.0001	0.2886775
-0.00001	0.2886754
-0.000001	0.2886752
0.000001	0.2886751
0.00001	0.2886749
0.0001	0.2886727
0.001	0.2886511

The limit appears to be approximately 0.2887.

$$\begin{aligned}
 \text{(c) } \lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} \right) &= \lim_{x \rightarrow 0} \frac{(3+x) - 3}{x(\sqrt{3+x} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} \\
 &= \frac{\lim_{x \rightarrow 0} 1}{\lim_{x \rightarrow 0} \sqrt{3+x} + \lim_{x \rightarrow 0} \sqrt{3}} && \text{[Limit Laws 5 and 1]} \\
 &= \frac{1}{\sqrt{\lim_{x \rightarrow 0} (3+x)} + \sqrt{3}} && \text{[7 and 11]} \\
 &= \frac{1}{\sqrt{3+0} + \sqrt{3}} && \text{[1, 7, and 8]} \\
 &= \frac{1}{2\sqrt{3}}
 \end{aligned}$$

38. We have $\lim_{x \rightarrow 1} (2x) = 2(1) = 2$ and $\lim_{x \rightarrow 1} (x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = 2$. Since $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x ,

$\lim_{x \rightarrow 1} g(x) = 2$ by the Squeeze Theorem.

60. Let $f(x) = \lceil x \rceil$ and $g(x) = -\lfloor x \rfloor$. Then $\lim_{x \rightarrow 3} f(x)$ and $\lim_{x \rightarrow 3} g(x)$ do not exist [Example 10]

but $\lim_{x \rightarrow 3} [f(x) + g(x)] = \lim_{x \rightarrow 3} (\lceil x \rceil - \lfloor x \rfloor) = \lim_{x \rightarrow 3} 0 = 0$.