Homework 5

Section 2.5:

10. (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.

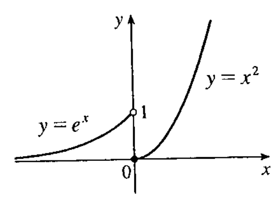
(b) Continuous, the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.

(c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values — at a cliff, for example.

(d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.

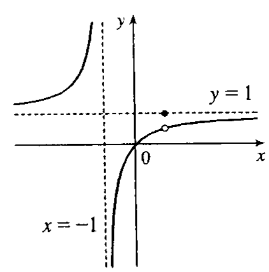
(e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

19. $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$



The left-hand limit of f at $a = 0$ is $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} e^x = 1$. The right-hand limit of f at $a = 0$ is $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2 = 0$. Since these limits are not equal, $\lim\limits_{x\to 0} f(x)$ does not exist and f is discontinuous at 0.

20. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$



$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x\to 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{x}{x+1} = \frac{1}{2},$$

but $f(1) = 1$, so f is discontinous at 1.

24. $f(x) = \dfrac{x^3 - 8}{x^2 - 4} = \dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \dfrac{x^2+2x+4}{x+2}$ for $x \neq 2$. Since $\lim\limits_{x\to 2} f(x) = \dfrac{4+4+4}{2+2} = 3$, define $f(2) = 3$.

Then f is continuous at 2.

Section 2.6:

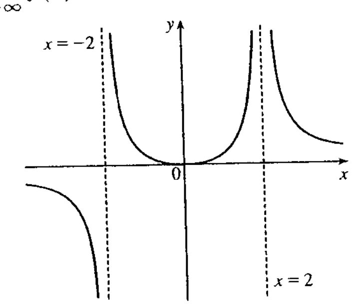
4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -1$ (c) $\lim_{x\to 0} g(x) = -\infty$

(d) $\lim_{x\to 2^-} g(x) = -\infty$ (e) $\lim_{x\to 2^+} g(x) = \infty$ (f) Vertical: $x = 0$, $x = 2$; horizontal: $y = -1$, $y = 2$

6. $\lim_{x\to 2} f(x) = \infty$, $\lim_{x\to -2^+} f(x) = \infty$,

$\lim_{x\to -2^-} f(x) = -\infty$, $\lim_{x\to -\infty} f(x) = 0$,

$\lim_{x\to\infty} f(x) = 0$, $f(0) = 0$



16. $$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{(1-x^2)/x^3}{(x^3-x+1)/x^3} = \lim_{x\to\infty} \frac{1/x^3 - 1/x}{1 - 1/x^2 + 1/x^3}$$

$$= \frac{\lim_{x\to\infty} 1/x^3 - \lim_{x\to\infty} 1/x}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} 1/x^2 + \lim_{x\to\infty} 1/x^3} = \frac{0-0}{1-0+0} = 0$$

18. $$\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5} = \lim_{x\to-\infty} \frac{(4x^3+6x^2-2)/x^3}{(2x^3-4x+5)/x^3} = \lim_{x\to-\infty} \frac{4+6/x-2/x^3}{2-4/x^2+5/x^3} = \frac{4+0-0}{2-0+0} = 2$$

34. Divide numerator and denominator by e^{3x}: $$\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = \lim_{x\to\infty} \frac{1-e^{-6x}}{1+e^{-6x}} = \frac{1-0}{1+0} = 1$$