Homework 8

Section 3.2:

2. Quotient Rule: $F(x) = \dfrac{x^4 - 5x^3 + \sqrt{x}}{x^2} = \dfrac{x^4 - 5x^3 + x^{1/2}}{x^2} \Rightarrow$

$$F'(x) = \frac{x^2(4x^3 - 15x^2 + \frac{1}{2}x^{-1/2}) - (x^4 - 5x^3 + x^{1/2})(2x)}{(x^2)^2} = \frac{4x^5 - 15x^4 + \frac{1}{2}x^{3/2} - 2x^5 + 10x^4 - 2x^{3/2}}{x^4}$$

$$= \frac{2x^5 - 5x^4 - \frac{3}{2}x^{3/2}}{x^4} = 2x - 5 - \tfrac{3}{2}x^{-5/2}$$

Simplifying first: $F(x) = \dfrac{x^4 - 5x^3 + \sqrt{x}}{x^2} = x^2 - 5x + x^{-3/2} \Rightarrow F'(x) = 2x - 5 - \frac{3}{2}x^{-5/2}$ (equivalent).

For this problem, simplifying first seems to be the better method.

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \Rightarrow g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

10. $J(v) = (v^3 - 2v)(v^{-4} + v^{-2}) \overset{\text{PR}}{\Rightarrow}$

$J'(v) = (v^3 - 2v)(-4v^{-5} - 2v^{-3}) + (v^{-4} + v^{-2})(3v^2 - 2)$

$= -4v^{-2} - 2v^0 + 8v^{-4} + 4v^{-2} + 3v^{-2} - 2v^{-4} + 3v^0 - 2v^{-2} = 1 + v^{-2} + 6v^{-4}$

20. $z = w^{3/2}(w + ce^w) = w^{5/2} + cw^{3/2}e^w \Rightarrow z' = \frac{5}{2}w^{3/2} + c\left(w^{3/2}\cdot e^w + e^w \cdot \frac{3}{2}w^{1/2}\right) = \frac{5}{2}w^{3/2} + \frac{1}{2}cw^{1/2}e^w(2w+3)$

22. $g(t) = \dfrac{t - \sqrt{t}}{t^{1/3}} = \dfrac{t}{t^{1/3}} - \dfrac{t^{1/2}}{t^{1/3}} = t^{2/3} - t^{1/6} \Rightarrow g'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{6}t^{-5/6}$

28. $f(x) = x^{5/2}e^x \Rightarrow f'(x) = x^{5/2}e^x + e^x \cdot \frac{5}{2}x^{3/2} = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x$ $\left[\text{or } \frac{1}{2}x^{3/2}e^x(2x+5)\right] \Rightarrow$

$f''(x) = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x + e^x\left(\frac{5}{2}x^{3/2} + \frac{15}{4}x^{1/2}\right) = \left(x^{5/2} + 5x^{3/2} + \frac{15}{4}x^{1/2}\right)e^x$ $\left[\text{or } \frac{1}{4}x^{1/2}e^x(4x^2 + 20x + 15)\right]$

32. $y = \dfrac{e^x}{x} \Rightarrow y' = \dfrac{x \cdot e^x - e^x \cdot 1}{x^2} = \dfrac{e^x(x-1)}{x^2}$.

At $(1, e)$, $y' = 0$, and an equation of the tangent line is $y - e = 0(x - 1)$, or $y = e$.

50. (a) $P(x) = F(x)\,G(x)$, so $P'(2) = F(2)\,G'(2) + G(2)\,F'(2) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2}$.

(b) $Q(x) = F(x)/G(x)$, so $Q'(7) = \dfrac{G(7)\,F'(7) - F(7)\,G'(7)}{[G(7)]^2} = \dfrac{1 \cdot \frac{1}{4} - 5 \cdot \left(-\frac{2}{3}\right)}{1^2} = \dfrac{1}{4} + \dfrac{10}{3} = \dfrac{43}{12}$