Homework 9

Section 3.3:

2. $f(x) = \sqrt{x}\sin x \quad \Rightarrow \quad f'(x) = \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x}\cos x + \dfrac{\sin x}{2\sqrt{x}}$

4. $y = 2\sec x - \csc x \quad \Rightarrow \quad y' = 2(\sec x\, \tan x) - (-\csc x\, \cot x) = 2\sec x\, \tan x + \csc x\, \cot x$

6. $g(\theta) = e^{\theta}(\tan\theta - \theta) \quad \Rightarrow \quad g'(\theta) = e^{\theta}(\sec^2\theta - 1) + (\tan\theta - \theta)e^{\theta} = e^{\theta}(\sec^2\theta - 1 + \tan\theta - \theta)$

8. $f(t) = \dfrac{\cot t}{e^t} \quad \Rightarrow \quad f'(t) = \dfrac{e^t(-\csc^2 t) - (\cot t)e^t}{(e^t)^2} = \dfrac{e^t(-\csc^2 t - \cot t)}{(e^t)^2} = -\dfrac{\csc^2 t + \cot t}{e^t}$

12. $y = \dfrac{\cos x}{1 - \sin x} \quad \Rightarrow$

$$y' = \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$

$$= \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

18. $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x} = \sec x\, \tan x$

22. $y = e^x \cos x \quad \Rightarrow \quad y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x) \quad \Rightarrow$ the slope of the tangent line at $(0, 1)$ is $e^0(\cos 0 - \sin 0) = 1(1 - 0) = 1$ and an equation is $y - 1 = 1(x - 0)$ or $y = x + 1$.

31. (a) $f(x) = \dfrac{\tan x - 1}{\sec x} \quad \Rightarrow$

$$f'(x) = \frac{\sec x(\sec^2 x) - (\tan x - 1)(\sec x \tan x)}{(\sec x)^2} = \frac{\sec x(\sec^2 x - \tan^2 x + \tan x)}{\sec^2 x} = \frac{1 + \tan x}{\sec x}$$

(b) $f(x) = \dfrac{\tan x - 1}{\sec x} = \dfrac{\dfrac{\sin x}{\cos x} - 1}{\dfrac{1}{\cos x}} = \dfrac{\dfrac{\sin x - \cos x}{\cos x}}{\dfrac{1}{\cos x}} = \sin x - \cos x \quad \Rightarrow \quad f'(x) = \cos x - (-\sin x) = \cos x + \sin x$

(c) From part (a), $f'(x) = \dfrac{1 + \tan x}{\sec x} = \dfrac{1}{\sec x} + \dfrac{\tan x}{\sec x} = \cos x + \sin x$, which is the expression for $f'(x)$ in part (b).