

In-Class Questions for April 16th

1. Which of the following questions does L'Hospital's rule apply to? If L'Hospital's applies, use it to calculate the limit; if not, calculate the limit using other rules.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x - 2}$

Well, we can compute this limit by directly plugging in 1. L'Hospital's rule is not necessary here because the limit is not of an indeterminate form. The only time we should use this method is when the limit produces $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^2 + 1}{x - 2} &= \frac{(1^2) + 1}{(1 - 2)} \\ &= \frac{2}{-1} \\ &= \boxed{-2}\end{aligned}$$

(b) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

For this limit, we notice that if we directly plug zero into the limit, we obtain the indeterminate form $\frac{0}{0}$, which is exactly the case when we should use L'Hospital's rule.

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin x}{x} &= \lim_{x \rightarrow 0} \frac{(\sin x)'}{(x)'} \\ &= \lim_{x \rightarrow 0} \frac{\cos x}{1} \\ &= \frac{\cos(0)}{1} \\ &= \frac{1}{1} \\ &= \boxed{1}\end{aligned}$$

$$(c) \lim_{x \rightarrow 1^+} \frac{x^2}{1-x}$$

It is not necessary to use L'Hospital's rule to calculate this limit because it would not be of an indeterminate form to plug in a number a little to the right of 1, which should give us a decent idea as to what the limit is doing as it approaches 1 from the right hand side.

$$\begin{aligned} \lim_{x \rightarrow 1^+} \frac{x^2}{1-x} &= \frac{(1.001)^2}{1-(1.001)} \\ &= \frac{1}{\text{small negative number}} \\ &= \boxed{-\infty} \end{aligned}$$

2. Calculate

$$\lim_{x \rightarrow 0^+} x \ln x$$

by rearranging it as a fraction and then using L'Hospital's (check that using it is appropriate!). Try both ways of rearranging it to see which one works best.

We must approach this problem in a different manner; L'Hospital's rule applies to ratios of functions, that is to say, we need to make $x \ln x$ look like one function is dividing the other. We can do this in two ways:

$$\begin{aligned} \lim_{x \rightarrow 0^+} x \ln x &= \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{x}{\frac{1}{\ln x}} \\ &= \lim_{x \rightarrow 0^+} \frac{(\ln x)'}{(\frac{1}{x})'} = \lim_{x \rightarrow 0^+} \frac{(x)'}{(\frac{1}{\ln x})'} \\ &= \lim_{x \rightarrow 0^+} \frac{(\frac{1}{x})}{(-\frac{1}{x^2})} = \lim_{x \rightarrow 0^+} \frac{(1)}{-\frac{(\frac{1}{x})}{(\ln x)^2}} \\ &= \lim_{x \rightarrow 0^+} \frac{-x^2}{x} = \lim_{x \rightarrow 0^+} -x(\ln x)^2 \\ &= \lim_{x \rightarrow 0^+} -x = \text{Something ugly doing it this way.} \\ &= \boxed{0} \end{aligned}$$

The first column of equations represents the first way we could have done this, and it actually gets us in the right direction. The second way, which is putting $\ln(x)$ on the bottom, is actually counter productive because we must use the quotient rule (or reciprocal rule) to calculate the lower derivative. It becomes messy, and we still can't calculate it.