In-Class Questions for April 20th

Below, you're given a number of word problems. Some of them are related rates, and some of them are optimization.

Part 1: For the first in-class work period, I just want you to set up the questions. This corresponds to Steps 1 through 4 of the appropriate algorithm. A couple of things I'm going to ask you to do:

1. Write down a description in words for each variable you define. (Do so even if you also have a diagram.)
2. If a variable is measuring a length in your diagram, be very clear what length it is. Please label it with a $\overbrace{\hphantom{xxx}}$ so I can see precisely what you mean.

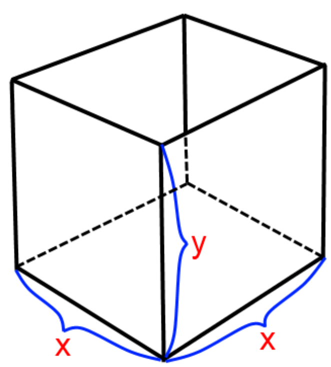
Part 2: For the optimization problems, do steps 5 and 6 of the algorithm (we'll do Step 7 next time.) Don't worry about the related rates ones – this is just supposed to be a reminder of how to set them up.

The Questions

1. Find the maximum volume of a box that can be constructed with 16 square feet of material, if its bottom must be square, and if its top is open.

   Solution:

   1. Here's the diagram:



   2. As labelled above, x is the length (and width) of the bottom of the box, and y is the height of the box. Also, let V be the volume of the box. We're maximizing V.

3. $V = x^2 y$, since the volume is just the product of the side lengths.
4. Since we have 16 square feet of material, the total surface area must be 16. The area of the bottom of the box is x^2, and the area of each of the four sides is xy. Thus, the relationship is

$$4xy + x^2 = 16$$

2. What is the maximum product of a number and 2 minus three times that number?

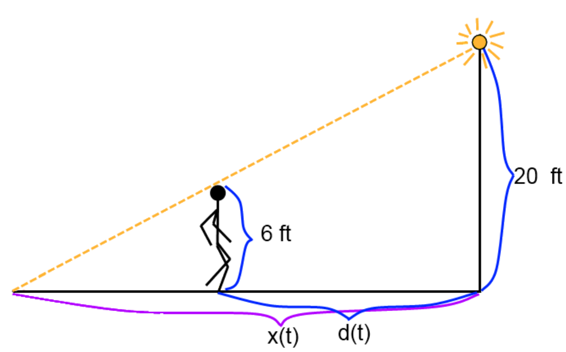
Solution:

1. No diagram needed.
2. Let x be the number in question. Let P be the product we're asked about.
3. Here, $P = x(2 - 3x)$.
4. No relationships needed – P is already in terms of one variable.

3. There is a streetlight on top of a 20 foot pole. A person who is 6 foot tall walks towards from the pole at 3 feet per second. How quickly is the tip of his shadow moving?

Solution:

1. Here's the diagram.



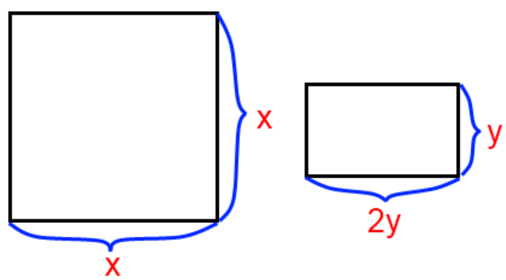
2. Here, $d(t)$ is the distance of the man from the pole, and $x(t)$ is the distance of the tip of his shadow from the pole.

3. Given: $d'(t) = -3$. The negative sign is there because he's moving towards the pole.
4. Need to find: $x'(t)$. This will be the velocity of the tip of the shadow, since $x(t)$ is the position of the tip with respect to the base of the streetlight.

4. We have a 10 foot wire. We split the wire into two pieces, and bend the first piece into a square, and the second piece into a rectangle one of whose sides is twice the other. What is the maximum combined area of the two shapes?

Solution:

1. Here's the diagram:



2. As labelled above, x is the side length of the square, whereas y and $2y$ are the side lengths of the rectangle. Furthermore, let A be the combined area of the two shapes.
3. $A = x^2 + 2y^2$.
4. Since the total wire used for the shapes should be 10 feet, the total perimeter is 10 feet. The perimeter of the square is $4x$, and the perimeter of the rectangle is $6y$. Thus, the relationship is

$$4x + 6y = 10$$