In-Class Questions for April 30th

1. Write down the following sums using sigma notation.

   (a) $1 + 2 + 3 + 4 + 5 + 6$

   $\sum_{i=1}^{6} i$

   (b) $4^2 + 5^2 + 6^2$

   $\sum_{i=4}^{6} i^2$

2.

$$\frac{1}{10}e^2 + \frac{1}{10}e^{\frac{21}{10}} + \frac{1}{10}e^{\frac{22}{10}} + \ldots + \frac{1}{10}e^{\frac{29}{10}}$$

   (a) Write down this sum using sigma notation.

   $(\frac{1}{10}) \sum_{i=20}^{29} e^{\frac{i}{20}}$

   (b) What integral does this sum approximate? Is it using left or right endpoints?

   This sum approximates the function $f(x) = e^x$ with 10 rectangles using left end points. $(\frac{1}{10})$ is the width of each rectangle.

3. If $\int_0^3 f(x)dx = -1$, $\int_3^4 f(x)dx = 4$, and $\int_0^2 g(x)dx = 2$, calculate:

   (a) $\int_0^4 f(x)dx$.

$$\begin{aligned}\int_0^4 f(x)dx &= \int_0^3 f(x)dx + \int_3^4 f(x)dx \\ &= (-1) + (4) \\ &= 3\end{aligned}$$

(b) $\int_0^4 (2f(x) - g(x) + 2)dx$.

$$\begin{aligned}
\int_0^4 (2f(x) - g(x) + 2)dx &= \int_0^4 (2f(x)) + \int_0^4 (-g(x)) + \int_0^4 (2)dx \\
&= \int_0^4 (2f(x)) + \int_0^4 (-g(x)) + \int_0^4 (2)dx \\
&= 2\int_0^4 (f(x)) + \int_0^4 (-g(x)) + (2(4-0)) \\
&= 2[\int_0^3 (f(x)) + \int_3^4 (f(x))] - \int_0^4 (g(x)) + (8) \\
&= 2[(-1) + (4)] - (2) + (8) \\
&= (12) - (2) + (8) \\
&= 18
\end{aligned}$$

Super cool!