2/1/'12 Solutions

Part 1

1. $\lim_{x \to 1} \dfrac{\sqrt{x+3}-2}{x-1}$

if substitute 1 $\dfrac{\sqrt{1+3}-2}{1-1} = \dfrac{0}{0}$ not good!

so multiply by conjugate

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{(x-1)} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \frac{(\sqrt{x+3})^2 + (-2)(2)}{(x-1)(\sqrt{x+3}+2)} = \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \frac{\overset{\text{cancel out}}{\cancel{(x-1)}}}{\cancel{(x-1)}(\sqrt{x+3}+2)}$$

$$\lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} \quad \text{substitute 1 in for } x = \lim_{x\to 1} \frac{1}{\sqrt{1+3}+2} = \boxed{\frac{1}{4}}$$

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2. $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$ where $f(x) = \dfrac{1}{x}$

rewrite ↑ this equation substituting $(2+h)$ for x & 2 for x

$$\lim_{h\to 0} \frac{\frac{1}{(2+h)} - \frac{1}{2}}{h}$$

find like denometers to add the top 2 fractions

$$\lim_{h\to 0} \frac{\frac{1}{2+h}\cdot\left(\frac{2}{2}\right) - \frac{1}{2}\left(\frac{2+h}{2+h}\right)}{h}$$

$$\lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h}$$

distribute

$$\lim_{h\to 0} \frac{\frac{2-2-h}{4+2h}}{h}$$

$$\lim_{h\to 0} \frac{\frac{-h}{4+2h}}{\left(\frac{h}{1}\right)}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{4+2h} \cdot \frac{1}{\cancel{h}}$$

h's cancel out

$$\lim_{h\to 0} \frac{-1}{4+2h}$$

plug in 0 for h

$$\lim_{h\to 0} \boxed{\frac{-1}{4}}$$

Part 2

$\boxed{1}$ find $\lim_{x \to 1} f(x)$ if

$$2x-1 \le f(x) \le x^2$$

for $0 \le x \le 3$. use picture to illustrate

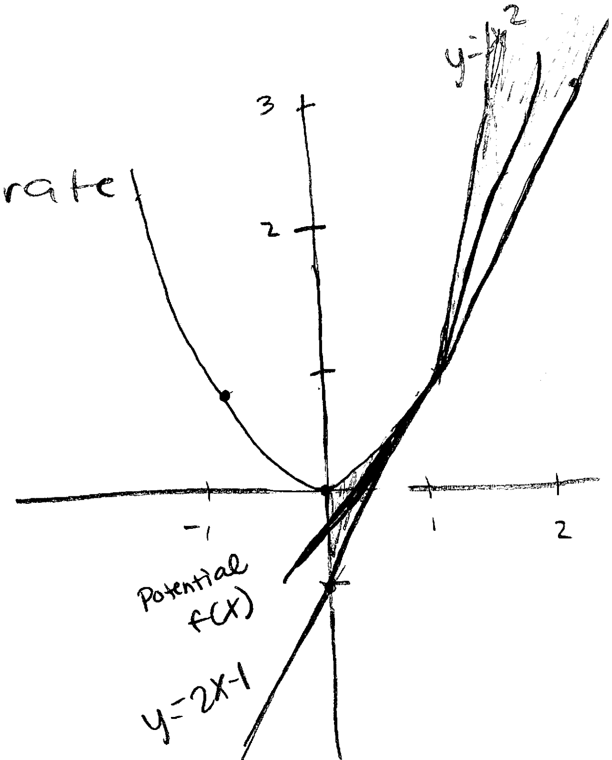
-Use squeeze theorem

$$\lim_{x \to 1} 2x-1 \le \lim_{x \to 1} f(x) \le \lim_{x \to 1} x^2$$

Plug in 1   Plug in 1

$$1 \le \lim_{x \to 1} f(x) \le 1$$

since both are = to 1 then $\lim_{x \to 1} f(x)$ $\boxed{1}$



The 2 functions both touch at (1,1). Since f(x) is defined as being inbetween these 2 functions (the shaded region), $\lim_{x \to 1} f(x)$ must be 1.

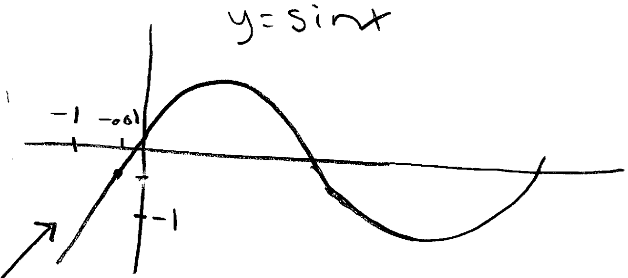
$\boxed{2.}$ calculate $\lim_{x \to 0^-} \frac{1}{x\sin(x)}$

: Plug in 0 $\lim_{x \to 0^-} \frac{1}{0\sin(0)} = \frac{1}{0}$ no good!

so pick a # close to 0 from negative side (like -.01)

$$\lim_{x \to 0^-} \frac{1}{x\sin(x)}$$

$$\lim_{x \to 0^-} \frac{1}{-.01\sin(-.01)}$$



look at graph at sin(-.01) y= small - #

negative x negative = positive $\lim_{x \to 0^-} \frac{1}{-.01 \text{ small negative \#}}$

$$\lim_{x \to 0^-} \frac{1}{\text{small positive \#}}$$

$$\lim_{x \to 0^-} \boxed{\infty}$$