GROUP WORK
13 FEB 2012

① Calculate $f'(x)$ given

$$f(x) = x^2 + \frac{1}{2\sqrt{x}} + e^{x-1}$$

$$f'(x) = [x^2]' + \left[\frac{1}{2\sqrt{x}}\right]' + \left[e^{x-1}\right]'$$

A derivative of a sum is the derivative of each of its terms

$$f'(x) = 2x + \frac{1}{2}\left[\frac{1}{\sqrt{x}}\right]' + e^{-1}\left[e^{x}\right]'$$

Derivative of x^2 is $2x$. pull out coefficients.

$$f'(x) = 2x + \frac{1}{2}\left[x^{-1/2}\right]' + e^{-1}(e^x)$$

$\frac{1}{\sqrt{x}} = x^{-1/2}$, and the derivative of e^x is itself, e^x.

$$f'(x) = 2x + \frac{1}{2}\left(\frac{-1}{2}\right)x^{-3/2} + e^{x-1}$$

Derivative of $x^{-1/2}$ is $\left(\frac{-1}{2}\right)x^{-3/2}$.

$$\boxed{f'(x) = 2x - \frac{1}{4\sqrt{x^3}} + e^{x-1}}$$

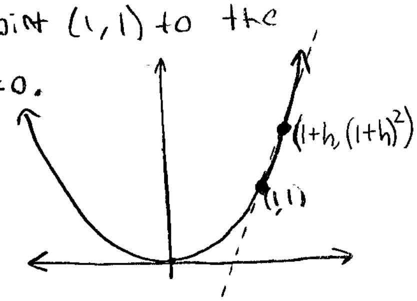
② Consider the parabola $f(x) = x^2$.

a) What does the quantity $\frac{(1+h)^2 - 1}{h}$ represent for this graph?

- Does this look familiar? Consider our limit definitions for the derivative at a point, a:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

It's similar to the second limit definition for the derivative, although that's not quite what this is describing. If we compare $\frac{(1+h)^2 - 1}{h}$ to $\frac{f(a+h) - f(a)}{h}$, then we see that $a = 1$ and $f(a) = f(1) = 1$. $\frac{(1+h)^2 - 1}{h}$ represents the slope of the line connecting the point $(1,1)$ to the point $(1+h, (1+h)^2)$. It is the slope of a secant line when $h \neq 0$.



b) Use your answer from (a) to explain why

$$f'(1) = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h}.$$

The slope of the tangent line at $x=1$ is equal to $\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$ because the slope of a secant line connecting points $(1,1)$ and $(1+h, f(1+h))$ becomes a better approximation as h becomes small. For small values of h, the point $(1+h, f(1+h))$ becomes very close to $(1,1)$. As h approaches zero, it is no longer an estimation of the slope of the tangent line; it <u>is</u> precisely the slope of the tangent line at $x=1$, or $f'(1)$.

c) $f(x) = x^2$; use the limit definition of the derivative <u>as a function</u> to calculate $f'(x)$.

Recall: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, so

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} =$$

$$\lim_{h \to 0} 2x + h \to \text{(Direct Substitution)} \to = \boxed{2x}.$$

- Refer to examples 2, 3, 4, and 5 on pages 155 – 158 (Chapter 2.8) in your book.