In Class Feb. 20th

Part 1:

1) a) $f(x) = x(1-x^2)$ calculate $f'(x)$ using product rule

$g(x) = x$ ... $g'(x)h(x) + g(x)h'(x) = f'(x)$

$g'(x) = 1$ $\quad 1(1-x^2) + x(-2x) = f'(x)$

$h(x) = (1-x^2)$

$h'(x) = (-2x)$ $\quad 1-x^2 + (-2x^2) = f'(x)$

$1-x^2-2x^2 = f'(x)$

$\boxed{1-3x^2 = f'(x)}$

b) $f(x) = x(1-x^2)$

$f(x) = x - x^3$ ← distribute

$\boxed{f'(x) = 1 - 3x^2}$

c) $1-3x^2 = 1-3x^2$ ✓

2.) Show $(fg)'(x) = f'(x)g'(x)$ is wrong by giving example.

$f(x) = 2x \quad g(x) = 3x^2$

$f'(x) = 2 \quad g'(x) = 6x$

Right way (Product Rule)

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

$(fg)'(x) = (2)(3x^2) + (2x)(6x)$

$(fg)'(x) = 6x^2 + 12x^2$

$(fg)'(x) = 18x^2$

Wrong way

$(fg)'(x) = f'(x)g'(x)$

$(fg)'(x) = 2(6x)$

$(fg)'(x) = 12x$

$\boxed{18x^2 \neq 12x}$

no equal!

Part 2

1.) Find instantaneous rate of change of f at $x=0$, if

$$f(x) = \frac{1+x}{e^x+1}$$

* key idea: instantaneous rate of change = slope of tangent line = derivative

$g(x) = (1+x)$ $\quad h(x) = e^x+1$

$g'(x) = 1$ $\quad h'(x) = e^x$

$$f'(x) = \frac{g'(x)h(x) - (g(x)h'(x))}{h(x)^2}$$

$$f'(x) = \frac{1(e^x+1) - ((1+x)(e^x))}{(e^x+1)^2}$$

$$f'(x) = \frac{e^x+1-(e^x+xe^x)}{(e^x+1)^2}$$

$$f'(x) = \frac{e^x+1-e^x-xe^x}{(e^x+1)^2}$$

$$\rightarrow f'(x) = \frac{1-xe^x}{(e^x+1)^2}$$

$$f'(0) = \frac{1-(0)e^0}{(e^0+1)^2}$$

$$f'(0) = \frac{1}{(1+1)^2}$$

$$f'(0) = \frac{1}{2^2}$$

$$\boxed{f'(0) = \frac{1}{4}}$$

2.) Find all points P at which tangent of $y = x^3 - 3x + 1$ is parallel to $y = 6x+8$.

Parallel = slopes are equal

$y = 6x+8$ $\quad y = x^3 - 3x + 1$

$y' = 6$ $\quad y' = 3x^2 - 3$

need to find x which these eqns are equal

$6 = 3x^2 - 3$

$+3 \quad +3$

$9 = 3x^2$

$\frac{9}{3} = \frac{3x^2}{3}$

$\sqrt{3} = \sqrt{x^2}$

$\pm\sqrt{3} = x$

now need to find y value at $x = \pm\sqrt{3}$

use $y = x^3 - 3x + 1$ & plug in x

$x = \sqrt{3}$

$y = (\sqrt{3})^3 - 3\sqrt{3} + 1$

$y = 3^{3/2} - 3\sqrt{3} + 1$

$y = 3\sqrt{3} - 3\sqrt{3} + 1$

$y = 1$ $\quad \boxed{(\sqrt{3}, 1)}$

$x = -\sqrt{3}$

$y = (-\sqrt{3})^3 + 3\sqrt{3} + 1$

$y = -3\sqrt{3} + 3\sqrt{3} + 1$

$y = 1$

$\boxed{(-\sqrt{3}, 1)}$