## In-Class Work Solutions for February 24th

**Part 1:**

1. Write the following expressions as single fractions, simplifying as much as possible. Make sure to only the rules learned in class, and state which rule you're using at each step!

   (a)
$$(x-1)\cdot\frac{1}{x^2}-\frac{2}{x}$$

   **Solution:**

   Using our rules,

$$\begin{aligned}
(x-1)\cdot\frac{1}{x^2}-\frac{2}{x} &= \frac{x-1}{1}\cdot\frac{1}{x^2}-\frac{2}{x} && \text{(Rule 4)}\\
&= \frac{x-1}{x^2}-\frac{2}{x} && \text{(Rule 2)}\\
&= \frac{x-1}{x^2}-\frac{2\cdot x}{x\cdot x} && \text{(Rule 1)}\\
&= \frac{x-1}{x^2}-\frac{2x}{x^2}\\
&= \frac{x-1-2x}{x^2} && \text{(Rule 3)}\\
&= \boxed{\frac{-x-1}{x^2}}
\end{aligned}$$

   (b)
$$\frac{e^x}{\cos(x)}+\sin(x)$$

   **Solution:**

   Again, using the rules,

$$\begin{aligned}
\frac{e^x}{\cos(x)}+\sin(x) &= \frac{e^x}{\cos(x)}+\frac{\sin(x)}{1} && \text{(Rule 4)}\\
&= \frac{e^x}{\cos(x)}+\frac{\sin(x)\cos(x)}{\cos(x)} && \text{(Rule 1)}\\
&= \boxed{\frac{e^x+\sin(x)\cos(x)}{\cos(x)}} && \text{(Rule 3)}
\end{aligned}$$

   (c)
$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

**Solution:**

Here, the easiest thing to do is the simplify the numerator first:

$$\begin{aligned} \frac{1}{x+h} - \frac{1}{x} &= \frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} && \text{(Rule 1)} \\ &= \frac{x-(x+h)}{x(x+h)} && \text{(Rule 3)} \\ &= \frac{x-x-h}{x(x+h)} = \frac{-h}{x(x+h)} \end{aligned}$$

Now, to finish by putting it all over $h$:

$$\begin{aligned} \frac{\frac{-h}{x(x+h)}}{h} &= \frac{\frac{-h}{x(x+h)} \cdot \frac{1}{h}}{h \cdot \frac{1}{h}} && \text{(Rule 1)} \\ &= \frac{\frac{-1}{x(x+h)}}{1} && \text{(Rules 1 and 2)} \\ &= \boxed{\frac{-1}{x(x+h)}} && \text{(Rule 4)} \end{aligned}$$

**Part 2:**

1. In each of the following calculations, explain what wrong 'rule' gets used. Demonstrate using numbers why the rule *doesn't* work.

   (a)

$$\frac{x^2 \sin(x) + \cos^2(x)}{e^x \sin(x)} = \frac{x^2 + \cos^2(x)}{e^x}$$

**Solution:**

Here, we're attempting to cancel the first term in the numerator with the denominator. Trying this with numbers, we get that

$$\frac{2+1}{4} \stackrel{?}{=} \frac{1+1}{2}$$

which states that $\frac{3}{4}$ is equal to 1 – clearly not true.

   (b)

$$\frac{2x^2}{\sin(x)} = \frac{2x^2}{\sin(x)} \cdot \frac{1}{\sin(x)}$$

**Solution:**

Here, we've kind of mixed up the rules for adding and multiplying fractions: so we've multiplied the numerators, but not the denominators (which is indeed what happens with addition!) Trying it:

$$\frac{3}{2} \stackrel{?}{=} \frac{1}{2} \cdot \frac{3}{2}$$

which states that $\frac{3}{2}$ is equal to $\frac{3}{4}$ – not true!

(c)

$$\frac{2x}{e^x \sin(x)} + \frac{x^2}{\cos(x)} = \frac{2x + x^2}{e^x \sin(x) + \cos(x)}$$

**Solution:**

Here, we're again confusing the rules for adding and multiplying fractions. We're trying to add fractions by adding numerators and denominators. Trying it:

$$\frac{1}{2} + \frac{1}{2} \stackrel{?}{=} \frac{2}{4}$$

This states that $1 = \frac{2}{4} = \frac{1}{2}$, which is clearly not true!

2. Formulate a rule for dividing fractions, using the rules from class. That is, simplify the following expression as a single fraction, then explain the rule in words:

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

**Solution:**

$$\begin{aligned}
\frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}} && \text{(Rule 1)} \\
&= \frac{\frac{ad}{bc}}{\frac{cd}{cd}} && \text{(Rule 2)} \\
&= \frac{\frac{ad}{bc}}{1} && \text{(Rules 1 and 4)} \\
&= \frac{ad}{bc} && \text{(Rule 4)}
\end{aligned}$$