GROUP WORK
27 FEB 2012

1) Calculate $f'(x)$, if

Chain Rule/Mult. Rule

$$f(x) = \sin(e^x - 1) + \sqrt{1 + x^2\sec(x)}$$
$$f'(x) = \cos(e^x-1)\cdot e^x + \frac{d}{dx}\left[(1+x^2\sec(x))\right]^{1/2}$$
$$f'(x) = \cos(e^x-1)e^x + \left(\frac{1}{2}\right)(1+x^2\sec(x))^{-1/2}\cdot\left[x^2\sec(x)\tan(x) + 2x\sec(x)\right]$$
$$\boxed{f'(x) = \cos(e^x-1)e^x + \frac{x\sec(x)\left[x\tan(x)+2\right]}{2\sqrt{1+x^2\sec(x)}}}$$

2) Consider the function $g(x) = \frac{1}{f(x)}$. (For a general $f(x)$, not the previous one).

a) Find an expression for $g'(x)$ in terms of $f(x)$ and $f'(x)$ using the quotient rule.

$$g(x) = \frac{1}{f(x)};$$
$$g'(x) = \frac{f(x)\frac{d}{dx}[1] - (1)f'(x)}{[f(x)]^2}$$
$$= \frac{f(x)(0) - f'(x)}{[f(x)]^2}$$
$$= \boxed{\frac{-f'(x)}{[f(x)]^2}}$$

(also called the Reciprocal Rule)

b) Find an expression for $g'(x)$ using the chain rule.

$$g(x) = \frac{1}{f(x)} = [f(x)]^{-1}$$
$$g'(x) = (-1)[f(x)]^{-2}\cdot f'(x)$$
$$= \boxed{\frac{-f'(x)}{[f(x)]^2}}$$

c) (a) + (b) match.

Further clarification for #1: derivative of $\sin(u)$, $u = e^x - 1 = \cos(u)\cdot du$
$= \underline{\cos(e^x-1)\cdot e^x}$.

derivative of $(1 + x^2\sec(x))^{1/2}$: $\frac{d}{dx}\left[u^{1/2}\right]$, $u = 1 + x^2\sec(x) = \underline{\frac{1}{2}u^{-1/2}}\cdot\underline{du}$

$\underline{du} = \left[x^2\sec(x)\right]' = (2x\sec(x) + x^2\sec(x)\tan(x))$ by mult. Rule.

So, putting things together:

$$f'(x) = \cos(e^x-1)e^x + \left(\frac{1}{2}\right)(1+x^2\sec(x))^{-1/2}(2x\sec(x) + x^2\tan(x)\sec(x))$$

first term (chain rule) + second term (chain rule, then mult. rule).