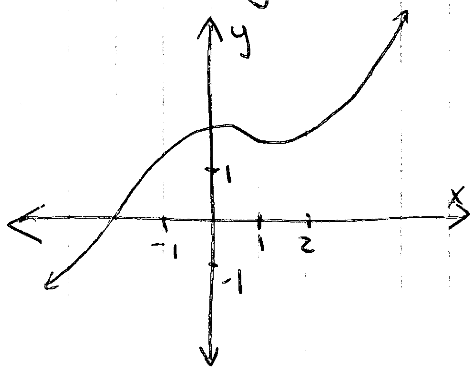
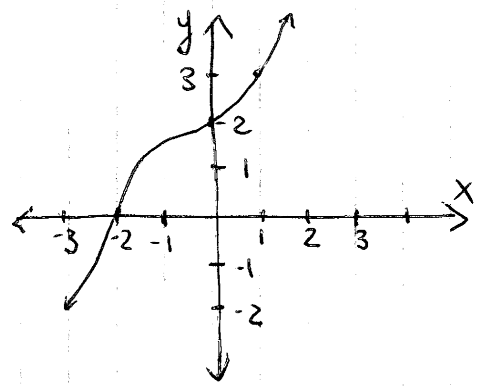
In-class Work: Friday, January 20th

1) Which of these two functions has an inverse, and why?





Solution

Only the function on the right has an inverse, since it passes the horizontal line test, while the one on the left does not.

2) For the $f(x)$ that has an inverse, without sketching f^{-1}, find

$$f^{-1}(2) \quad \text{and} \quad f^{-1}(3)$$

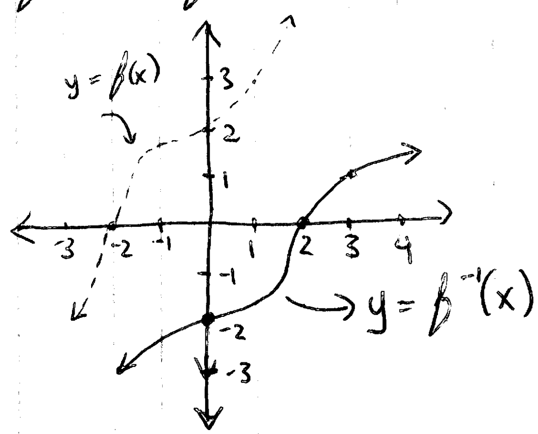
Solution By definition, $f^{-1}(x)$ is the value that f sends to x. Thus, to find $f^{-1}(2)$, we find the x-value that corresponds to the y-value 2. Since the graph has the point $(0, 2)$,

$$\boxed{f^{-1}(2) = 0}$$

Similarly, $\boxed{f^{-1}(3) = 1}$

3) Sketch $f^{-1}(x)$

Solution The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected across the line $y = x$. Thus, here's the graph of $f^{-1}(x)$, along with a dotted line graph of $f(x)$ for clarity:



4) (BONUS) What's $f^{-1}(f(x))$?

Solution By definition,

$f^{-1}(f(x))$ is the value that f sends to $f(x)$.

Clearly f sends x to $f(x)$ by definition.

Therefore,

$$\boxed{f^{-1}(f(x)) = x}$$

(You may have to mull this over before it makes perfect sense!)

Second In-class Work Period

1) Find $\log_4 16$, $\log_2 1$, $\log_3 \frac{1}{9}$

Solution

By definition, $\log_a x$ is the power a has to be raised to in order to get x.

Therefore:

- $\log_4 16$ is the power 4 has to be raised to in order to get 16. Since

$$4^2 = 16,$$

$$\boxed{\log_4 16 = 2}$$

- $\log_2 1$ is the power 2 has to be raised to in order to get 1. Since

$$2^0 = 1,$$

$$\boxed{\log_2 1 = 0}$$

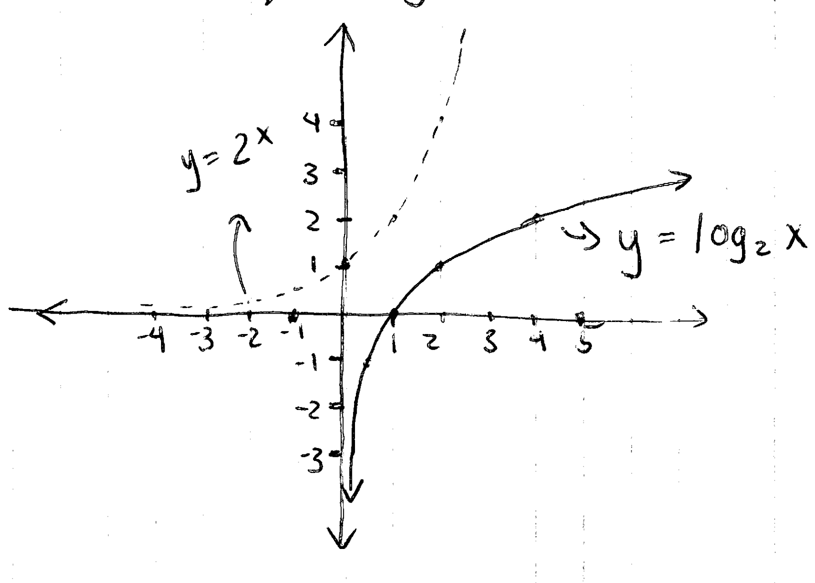
- $\log_3 \frac{1}{9}$ is the power 3 has to be raised to in order to get $\frac{1}{9}$. Since

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9},$$

$$\boxed{\log_3 \frac{1}{9} = -2}$$

2) Sketch a graph of $\log_2 x$.

<u>Solution</u> By definition, $\log_2 x$ is the inverse of 2^x. Thus, its graph can be obtained by reflecting 2^x across $y = x$.



Here are the graphs of $y = 2^x$ and $y = \log_2 x$, with $y = 2^x$ in dotted line.