Monday, January 23

First In-Class Work Period

1) Simplify the following as much as possible:

$$\ln\left(\frac{e^2 \cdot (a+b)^3}{3}\right)$$

Solution:

$$\ln\left(\frac{e^2 (a+b)^3}{3}\right) = \ln(e^2) + \ln\left(\frac{(a+b)^3}{3}\right) =$$

$$= 2\ln(e) + \ln((a+b)^3) - \ln(3)$$

$$= \boxed{2 + 3\ln(a+b) - \ln(3)}$$

Note that $\ln(e) = 1$ since $\ln(e) = \log_e e$, and $\ln(a+b)$ does not simplify further.

2) Solve following equation for x in terms of y:

$$e^x = (2+y)e^x + y^2$$

Solution Start by isolating the exponential e^x:

$$e^x = (2+y)e^x + y^2$$

$$\Rightarrow e^x = 2e^x + ye^x + y^2$$

$$\Rightarrow e^x - 2e^x - ye^x = y^2$$

$$\Rightarrow e^x(-1-y) = y^2$$

$$\Rightarrow e^x = \frac{y^2}{-1-y}$$

Now, take ln of both sides:

$$\ln(e^x) = \ln\left(\frac{y^2}{-1-y}\right) = \ln(y^2) - \ln(-1-y) =$$
$$= 2\ln(y) - \ln(-1-y)$$
$$\Rightarrow \boxed{x = 2\ln(y) - \ln(-1-y)}$$

Second In-Class Work Period

1) Find the formula for f^{-1}, if
$$f(x) = \frac{x}{1+x}$$

Solution:

Step 1: Write
$$y = \frac{x}{1+x}$$

Step 2: Solve for x:
$$(1+x) \cdot y = \frac{x}{1+x} \times (1+x)$$
$$\Rightarrow (1+x)y = x$$
$$\Rightarrow y + xy = x$$
$$\Rightarrow y = x - xy = x(1-y)$$
$$\Rightarrow \frac{y}{1-y} = x$$

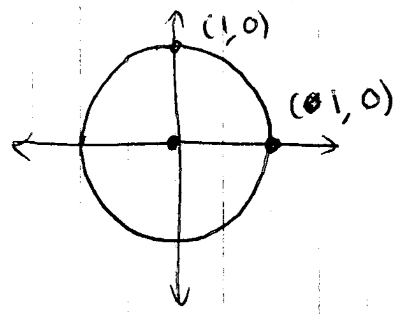
Step 3: Swap x and y:
$$y = \frac{x}{1-x}$$

Thus, $\boxed{f^{-1}(x) = \frac{x}{1-x}}$

2) What are $\sin^{-1}(0)$ and $\sin^{-1}(1)$?

<u>Solution</u>

By definition, $\sin^{-1}(0)$ is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is 0. Using the unit circle, we can see that this angle is 0. Thus,



$$\boxed{\sin^{-1}(0) = 0}$$

Similarly, $\sin^{-1}(1)$ is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is 1. Again, the unit circle shows that this angle is $\frac{\pi}{2}$, so

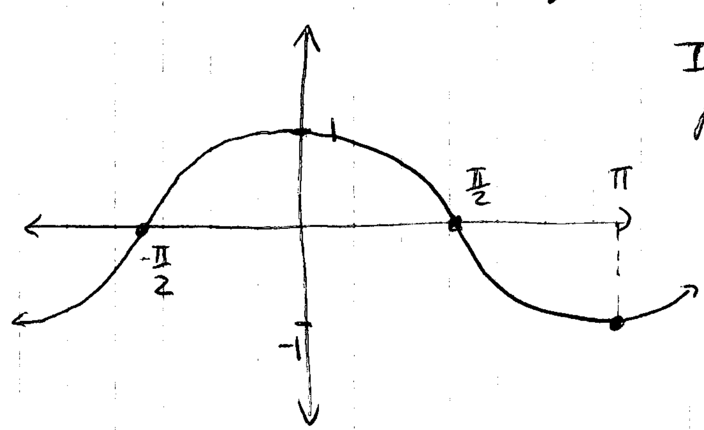
$$\boxed{\sin^{-1}(1) = \frac{\pi}{2}}$$

3) What interval should we restrict $\cos(x)$ to in order to be able to define its inverse? Use this to sketch $\cos^{-1}(x)$

<u>Solution</u> We need an interval on which

a) $\cos(x)$ is one-to-one
b) $\cos(x)$ takes on all the values in its range, which is $[-1, 1]$.

Hee's a sketch of $\cos(x)$:



It should be clear from the picture that two potential intervals are $[0, \pi]$ and $[-\pi, 0]$.

Conventionally, we pick $[0, \pi]$.

Thus, $\cos^{-1}(x)$ is really the inverse of $\cos(x)$ restricted to $[0, \pi]$, which has the following graph: on the left. And reflecting across $y = x$, we get the graph of $\cos^{-1}(x)$: