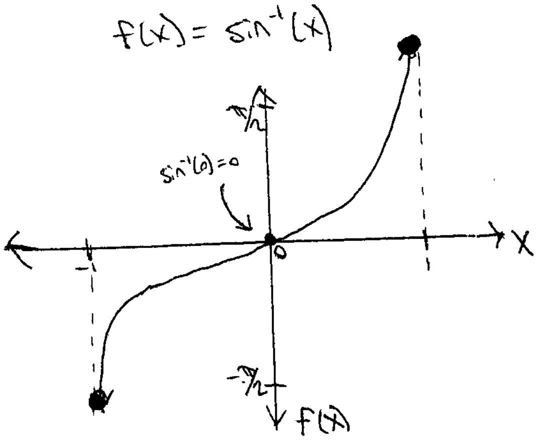
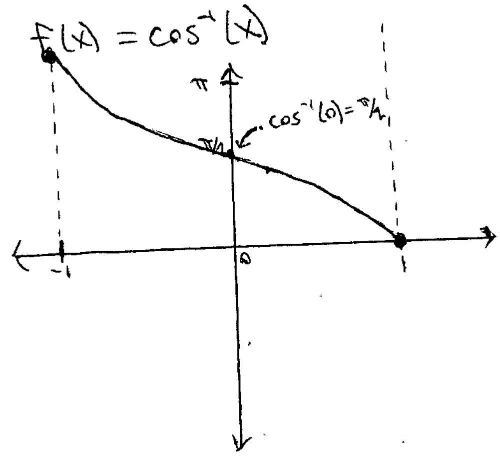
GROUP WORK 25 JAN 2012 (Wednesday)

① $\sin^{-1}(0) - 3\cos^{-1}(0)$

$\sin^{-1}(0) = 0$
$\cos^{-1}(0) = \frac{\pi}{2}$, so $\sin^{-1}(0) - 3\cos^{-1}(0) = 0 - 3\left(\frac{\pi}{2}\right) = \boxed{-\frac{3\pi}{2}}$

For reference, these are the graphs of $\sin^{-1}(x)$ and $\cos^{-1}(x)$:





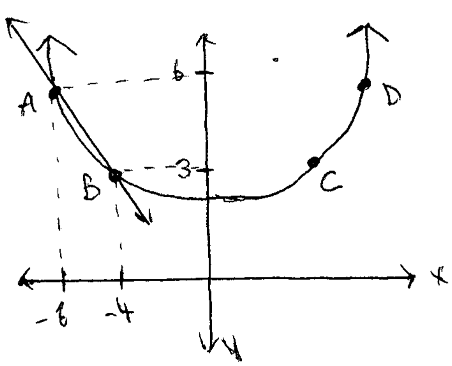
Recall, to plot the inverses of $\sin(x)$ and $\cos(x)$, we must restrict the domain for our graphs to pass the horizontal line test.

For $\sin(x)$, our domain is restricted from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and our range is thereby $[-1, 1]$. Notice how the domain and range of $\sin(x)$ switched when we plotted $\sin^{-1}(x)$.

Likewise, for $\cos(x)$, our domain is restricted to $[0, \pi]$, and thereby our range is $[-1, 1]$. When we plot $\cos^{-1}(x)$, the domains and range flipped yet again.

Cool, huh?

②



- Positive slope at points C+D.
- Negative slope at points A+B.
- The slope is maximum at point D. (it is nonnegative and larger than at point C).
- Slope of the secant connecting A+B?

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}} = \frac{3-6}{-4-(-6)} = \boxed{\frac{-3}{2}}$

A$(-6, 6)$
B$(-4, 3)$

25 JAN 2012 (Wednesday)

① Use the slope of the secants with $h = 0.1$ and -0.1 to guess the slope of the tangent at $y = x^2$ at the point $(2,4)$.

Recall: Our formula for finding the slope of a secant line arises from the mantra "rise over run," which uses the x and y values from two points to obtain the slope.

$$\frac{y_2 - y_1}{x_2 - x_1}$$ → Alter the formula, since we know that $y = f(x)$.

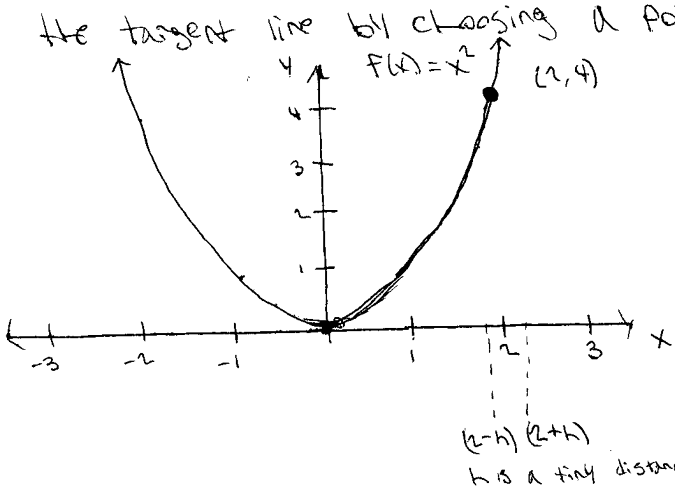
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$ Now, let's let x_2 represent a point a distance h away from x_1.

(Plug in (x_1+h) where you see x_2.)

(x_1 drops out of denominator)

$$\frac{f(x_1+h) - f(x_1)}{(\cancel{x_1}+h) - \cancel{x_1}} = \boxed{\frac{f(x_1+h) - f(x_1)}{h}}$$

So, we can use the formula $\frac{f(x+h) - f(x)}{h}$ to estimate the slope of the tangent line by choosing a point relatively close to $(2,4)$ on the graph.



Note: Choosing a smaller value for h will give a better approximation!!
$h = 0.001$ is better than $h = 0.1$.

$h = 0.1$, $\frac{f(x+0.1) - f(x)}{(0.1)}$ at point $(2,4)$ $= \frac{f(2+0.1) - f(2)}{(0.1)} = \frac{f(2.1) - f(2)}{(0.1)}$

$$= \frac{(2.1)^2 - (2)^2}{(0.1)} \approx \boxed{4.1}$$

$h = -0.1$, $\frac{f(x+(-0.1)) - f(x)}{(-0.1)} = \frac{f(2-0.1) - f(2)}{(-0.1)} = \frac{(1.9)^2 - 2^2}{-0.1} = \frac{-0.39}{-0.1} = \boxed{3.9}$

As you can see, these approximations of the slope of the tangent line are suspiciously close to 4. It is the right answer, actually.

GROUP WORK #2
25 JAN. 2012 (Wednesday)

③ Estimate the slope of the tangent line at the point (1,1) on the graph of $f(x) = \frac{1}{x}$.

Note: It isn't truly necessary to have a graph to do this calculation, and if you practice your algebra to convince yourself of this concept, then you will feel very prepared for similar questions in the future.

- We say the slope of the line connecting two points may be found by calculating the "rise over run" of the graph:

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

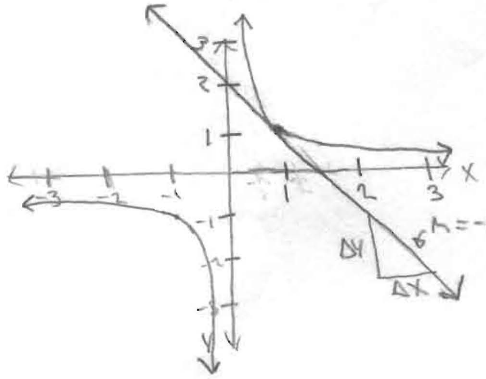
- Suppose we choose a point (x_2, y_2) on the graph of $f(x) = \frac{1}{x}$ to be very close to (x_1, y_1). Then, we get a decent approximation of the slope of the tangent line passing through (x_1, y_1).

- In our case, (x_1, y_1) is the point $(1,1)$. Look: $f(x) = \frac{1}{x}$; $f(1) = \frac{1}{(1)} = \boxed{1}$.

- Formula to approximate the slope of the tangent line:

$$m = \frac{f(x+h) - f(x)}{h}.$$

Let's estimate the slope using these values for h: 0.1, −0.1, 0.01, −0.01.



$$m = \frac{f(1+.1) - f(1)}{(.1)} = \frac{\left(\frac{1}{1.1}\right) - (1)}{(.1)} = \boxed{-0.91}, \quad h = 0.1$$

$$m = \frac{f(1-.1) - f(1)}{(-.1)} = \frac{\left(\frac{1}{0.9}\right) - 1}{(-.1)} = \boxed{-1.11}, \quad h = -0.1$$

$$m = \frac{f(1+.01) - f(1)}{(.01)} = \frac{\left(\frac{1}{1.01}\right) - 1}{(.01)} = \boxed{-0.99}, \quad h = 0.01$$

$$m = \frac{f(1-.01) - f(1)}{(-.01)} = \frac{\left(\frac{1}{0.99}\right) - 1}{-(.01)} = \boxed{-1.01}, \quad h = -0.01$$

- Note: As $|h|$ becomes very close to zero, our approximation approaches the real value for the slope of the tangent line. This is a fundamental result in calculus.

- For kicks, let $h = 0.00001$.

$$\frac{f(1 + .00001) - f(1)}{(.00001)} = \frac{\left(\frac{1}{1.00001}\right) - 1}{(.00001)} = \boxed{-0.99999},$$

which is remarkably close to $\boxed{-1}$.