GROUP WORK

PART I

2 MAR 2012

1) Consider the curve $x^2+y^2=25$.

(a) Find the slope of the tangent to the curve at $(4,-3)$ using implicit differentiation.

$$\frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[y^2\right]=\frac{d}{dx}\left[25\right]$$

$$2x + 2y(y') = 0$$

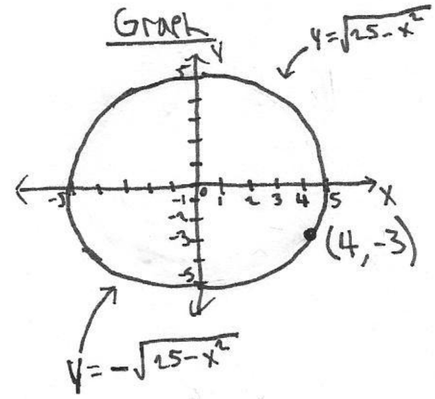
$$2y(y') = -2x$$

$$y' = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$$; Plug in $(4,3) \to y'(4) = -\frac{(4)}{(-3)} = \boxed{\frac{4}{3}}$

(b) Find the slope of the tangent to the curve at $(4,-3)$ by ~~the~~ first solving for y and doing normal differentiation. Make sure your answers to (a) and (b) match.

As we know, the curve $x^2+y^2=25$ represents a circle of radius, $r=5$, centered at the origin: As we can see, the point $(4,-3)$ lies on the bottom half of the circle. First, we solve for y in terms of x as we would normally do to ~~take~~ take the derivative.

Graph



$$x^2+y^2=25$$

$$y^2 = 25-x^2$$

$$y = \pm\sqrt{25-x^2}.$$

The positive half of the circle is given by the positive part of the square root, whereas the negative part is given by $y=-\sqrt{25-x^2}$. We're interested in taking the derivative of the curve that represents the bottom half of $x^2+y^2=25$.

So, $y = -\sqrt{25-x^2}$

$$y' = -\left(\frac{1}{2}\right)(25-x^2)^{-1/2}(-2x)$$

$$= \frac{-(-2)x}{(2)\sqrt{25-x^2}}$$

$$= \frac{x}{\sqrt{25-x^2}}.$$

$$y'(4) = \frac{(4)}{\sqrt{25-(4)^2}} = \frac{4}{\sqrt{9}} = \boxed{\frac{4}{3}}$$

Our results from (a) and (b) match.

PART 2

1) Consider the curve $x^2 - xy - 2^{y+1} = 0$

(a) Calculate y' in terms of x and y.

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[xy] - \frac{d}{dx}[2^{y+1}] = \frac{d}{dx}[0]$$

$$2x - [x(y)' + y(x)'] - 2^{y+1}\ln(2)\,y' = 0$$

$$2x - xy' - y - 2^{y+1}\ln(2)\,y' = 0$$

$$2x - y = xy' + 2^{y+1}\ln(2)\,y'$$

$$2x - y = y'(x + 2^{y+1}\ln(2))$$

$$\boxed{y' = \frac{2x - y}{x + 2^{y+1}\cdot\ln(2)}}$$

(b) Find the points on the curve with y-coordinate 2.

$$x^2 - xy - 2^{y+1} = 0, \quad y = 2.$$

$$x^2 - x(2) - 2^{(2)+1} = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4;\ x = -2.$$

(c) Find the slopes of the tangents to the curve at the points you found in (b).

points: $(4,2)$; $(-2,2)$.

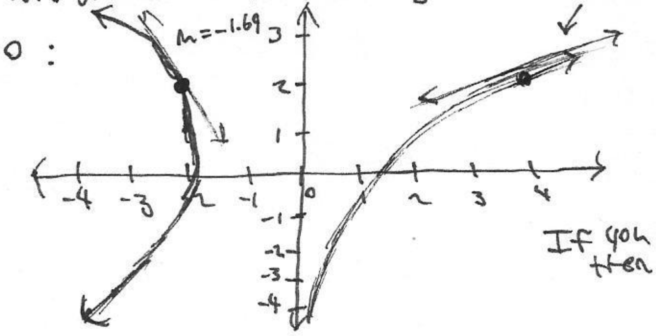
$$y'(4) = \frac{2(4) - (2)}{(4) + 2^{(3)}\ln(2)} = \frac{6}{4 + 8\ln(2)} = \boxed{\frac{3}{2 + 4\ln(2)}} \approx \boxed{0.629}$$

$$y'(-2) = \cancel{\frac{2(-2) - (-2)}{(-2) + 2^{(3)}}} = \frac{2(-2) - (2)}{(-2) + 2^{(3)}\ln(2)} = \frac{-6}{-2 + 8\ln(2)} = \boxed{\frac{-3}{-1 + 4\ln(2)}} \approx \boxed{-1.692}$$

This is why we can't have nice things.

GRAPH OF $x^2 - xy - 2^{y+1} = 0$:

(Poorly Drawn)



If you want a good graph, then try Wolfram Alpha.

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